ROOT LOCUS ANALYSIS WITH SPECIAL PARTITIONING

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SPECIAL PARTITIONING

by

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ABSTRACT

Root locus techniques are used infrequently in the design of tachometer and acceleration feedback compensation, for feedback control systems. A root locus stability criterion is discussed, which has the capability of handling more than one variable coefficient in the characteristic equation. This stability criterion is applied to the analysis and design of tachometer and acceleration feedback compensation. The design of cascade compensation is attempted, and the difficulties of designing this type of compensation with the root locus technique are discussed. The root locus stability criterion is found to be a useful tool for designing tachometer and acceleration feedback compensation for third order systems, and for certain fourth order systems.

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1. Introduction.

Tachometer and/or acceleration feedback are commonly used to compensate feedback control systems. This compensation is frequently designed from a Bode plot, less frequently root locus techniques are used. Tachometer and acceleration feedback, used together, involve two variable coefficients in the system characteristic equation, and neither method is really convenient.

A simple stability test, using root locus methods, has been presented by K. W. Han and G. J. Thaler which has the additional advantage of being capable of handling multiple adjustable coefficients in the characteristic equation. [1] This method can give considerable information about the effects of tachometer and acceleration feedback on a system, and some insight as to the reasons behind these effects.

Briefly, this root locus stability test involves partitioning the characteristic equation of the closed-loop system into even and odd parts, such that

$$F(S) = F_e(S) + F_o(S) = 0$$

and by then dividing through by $F_o(S)$ to get a proper root locus form

$$F_e(S)/F_o(S) = -1.$$

In general, this form of the characteristic equation can be written as

$$\frac{A(\text{ even polynomial })}{S(\text{ even polynomial })} = -1$$

where A is a gain constant.

It is known that all roots of the numerator and denominator polynomials will be on the imaginary axis of the S - plane. The only exception is the possible case of two pairs of complex roots with identical imaginary parts, and with the real part of one pair being the

negative of the real part of the other pair, as shown in Figure 1(b).

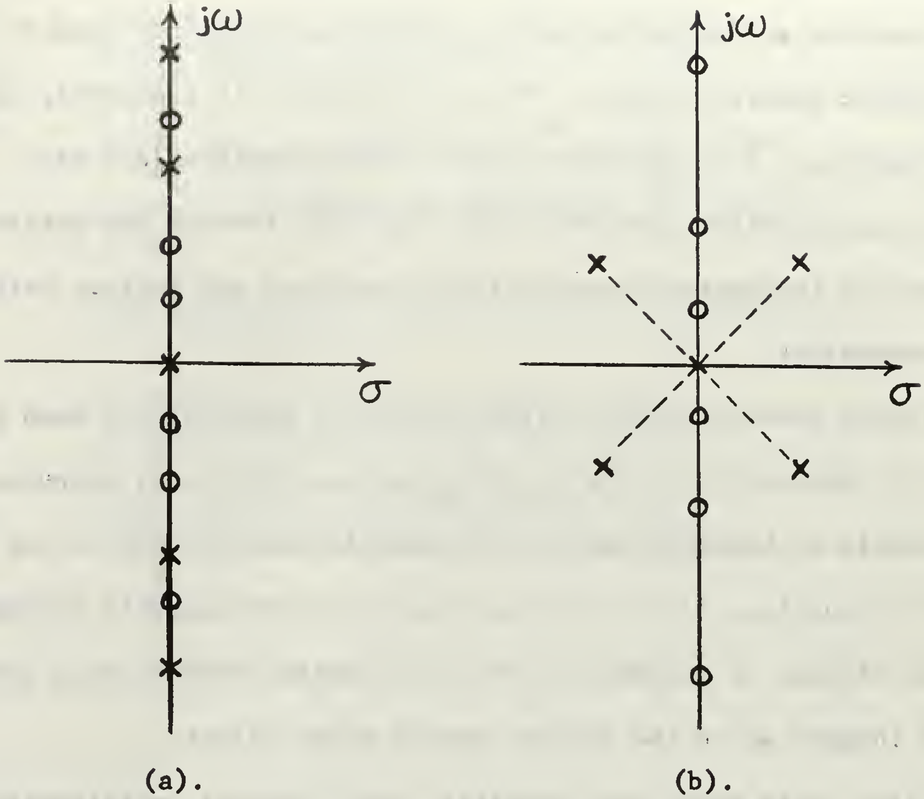


Fig. 1. Possible pole-zero distributions. (a). Unstable system with all roots on imaginary axis. (b). System with complex roots.

It is shown that for the system to be stable, the poles and zeros of the system must lie on the imaginary axis, and must be in the following, alternating sequence

$$P_0 < Z_1 < P_1 < Z_2 < P_2 < Z_3 \ \ldots$$

where P_0 is at the origin of the S-plane. A system with any other arrangement of poles and zeros is unstable.

Figure 2 shows a typical root locus for a stable system. In general, the loci will consist of lobes extending into the left hand plane. For any sequence of poles and zeros other than that specified, one or more of the root loci lobes will be completely in the right half plane.

These lobes may be roughly approximated by semi-circles, so that the maximum real part is approximately

$$\sigma = -\tfrac{1}{2} \left| P - Z \right| .$$

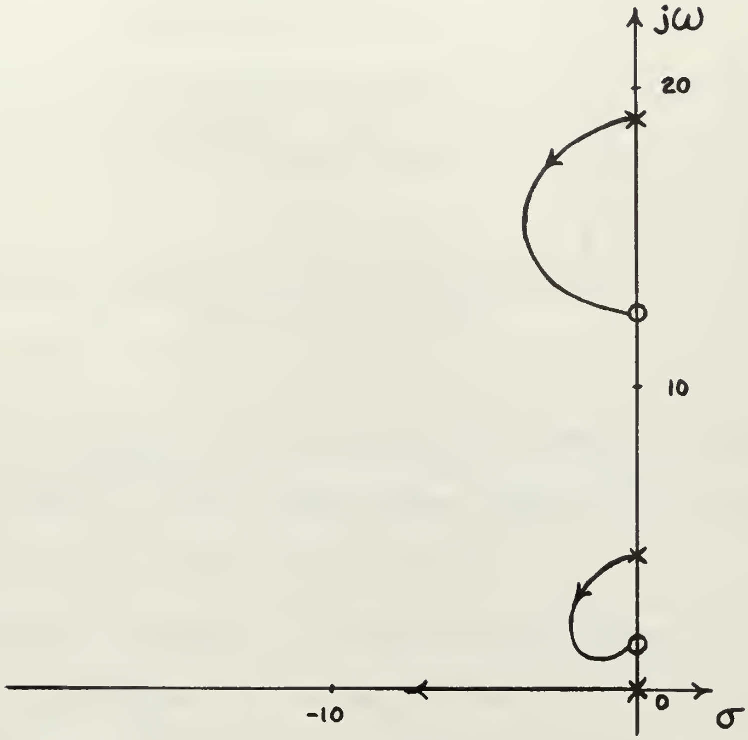


Fig. 2. Root loci for stable 5^{th} order system

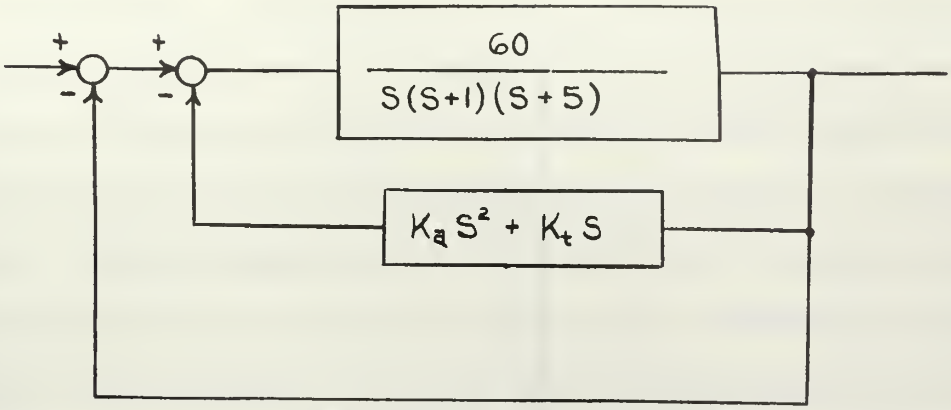
For purposes of design, ω_n is approximately bounded by the magnitudes of the poles and zeros for each lobe, and the maximum possible ζ can be approximated by constructing a line from the origin tangent to each lobe.

This thesis will apply the above root locus methods to the analysis of tachometer and acceleration feedback compensation of third, fourth, fifth and sixth order feedback control systems. Limitations to the ability of tachometer and acceleration feedback compensation to stabilize fifth and sixth order systems is presented. A design technique is discussed and applied to the design of tachometer and acceleration

feedback compensation of example control systems. The method is also used to design cascade compensation for a system.

The system performances, predicted by the design technique, are compared to the results of digital computer simulations of the example systems.

2. Stability analysis of third order system with tachometer and acceleration feedback compensation.



The effects of tachometer and acceleration feedback compensation on the third order feedback control system shown above are to be analyzed by means of the root locus stability criterion.

The characteristic equation of the closed loop system is

$F(s) = s^3 + (6 + 60K_a)s^2 + (5 + 60K_t)s + 60 = 0$.

Partitioning this equation into even and odd parts yields

$$F_e(s) = (6 + 60K_a)s^2 + 60$$

$$F_o(s) = s^3 + (5 + 60K_t)s \ .$$

The proper root locus form is obtained by forming $F_e(s)/F_o(s)$, which gives

$$\frac{(6 + 60K_a)[s^2 + 60/(6 + 60K_a)]}{s[s^2 + (5 + 60K_t)]} = -1 \, .$$

Setting $K_a = K_t = 0$ yields the basic system, which has the pole-zero distribution shown in Figure 3. It is apparent from the order of the poles and zeros that the system is unstable without compensation. In order to stabilize the system, it is necessary to interchange the order

of P_1 and Z_1. P_1 and Z_1 are called the critical pole and zero, as the stability of the system depends upon their locations on the imaginary axis.

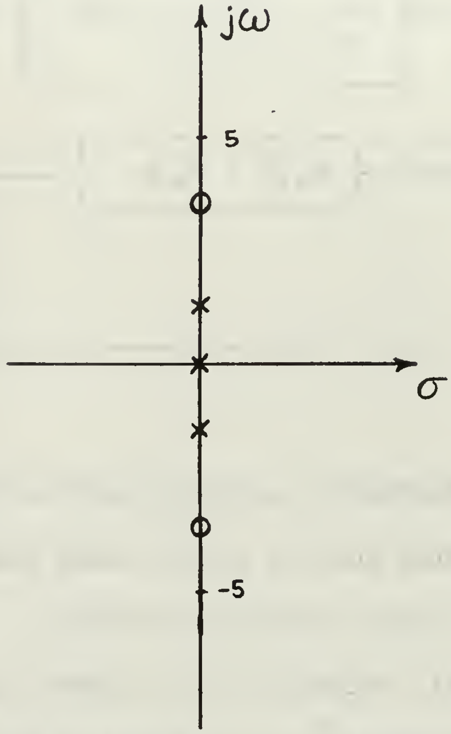


Fig. 3. Pole-zero plot for 3[rd] order system

Interchanging P_1 and Z_1 can be accomplished in one of three ways, (a), by moving the zero towards the origin, (b), by moving the pole away from the origin, or (c), by a combination of movements. Inspection of the even and odd parts of the characteristic equation shows that the second derivative acceleration feedback appears only in the even part, while the first derivative tachometer feedback appears only in the odd part. Therefore, adding acceleration feedback will cause only Z_1 to move. Increasing K_a, the acceleration feedback gain constant, will move Z_1 toward the origin, as required for stability. Likewise, adding tachometer feedback will cause P_1 to move away from the origin. Therefore, the system may be stabilized by adding acceleration feedback alone, tachometer feedback alone, or by a combination of tachometer and

acceleration feedback.

At this point, the effect of the various compensation schemes should be considered. The location of the root on the root locus lobe will, in general, depend upon the separation between the pole and zero, and upon the root locus gain constant. Low root locus gain causes the root to be located near the pole, high root locus gain moves the root nearer to the zero. Greater separation between the pole and zero causes the lobe to enlarge, and the root will not move as far, relatively, around the lobe for the same value of root locus gain. The greater separation also allows a larger maximum possible value of ζ . The possible values of ω_n are approximately bounded by the locations of the pole and zero.

The root locus gain constant, for the third order system, is the coefficient of the S^2 term. This coefficient is dependent on the acceleration feedback gain. An increase in the acceleration feedback gain raises the root locus gain, and moves the root of the system nearer to the zero. In effect, large values of acceleration feedback gain make ω_n approximately equal to the frequency of the zero, and decreases ζ . For these reasons, it may not be desirable to use large values of acceleration feedback gain in compensation of a third order system. On the other hand, large values of tachometer feedback gain tend to increase the possible ω_n , and may decrease ζ as the system root moves nearer to the pole. With a combination of tachometer and acceleration feedback, however, a satisfactory combination of ζ and ω_n should be attainable from a third order system.

In order to demonstrate the effects of varying tachometer feedback gain, acceleration feedback gain and system gain, the curves of Figures 4 and 5 are drawn. These curves are generated by considering the odd and even parts of the characteristic equation separately.

To determine the movements of the zero, consider $F_e(s)$.

$$(KK_a + 6)s^2 + K = 0$$

Partitioning this equation to separate the variable parameters, and putting it in root locus form yields

$$\frac{KK_a s^2}{6(s^2 + K/6)} = -1 .$$

By calculating the root locus for this equation, points may be generated for a curve showing the movement of the zero (Z_1) for varying KK_a. Various values of K can be substituted to give a family of curves describing the movements of Z_1 while varying K and KK_a.

To determine the movements of the pole (P_1), consider $F_o(s)$.

$$s^3 + (5 + KK_t)s = 0$$

Partitioning this equation as before yields

$$\frac{KK_t}{s^2 + 5} = -1.$$

A single curve may be generated, from the root locus calculations, showing the movement of P_1 for varying KK_t.

If $K_a = 0$, the even part is partitioned to give

$$\frac{K}{6s^2} = -1 .$$

A single curve may be generated to show the movement of Z_1 for varying K. If $K_t = 0$, P_1 will not change its location with variations in K or KK_a.

As stated before, the curves in Figures 4 and 5 were generated by calculating root loci and plotting the movements of P_1 and Z_1 as K, KK_t

and KK_a were varied. This would not be necessary, however, as points on the curves could be generated directly for this simple system. The location of P_1 varies as $(KK_t + 5)^{\frac{1}{2}}$, and the location of Z_1 varies as $[K/(KK_a + 6)]^{\frac{1}{2}}$.

For any given system, values of K_t and K_a could be found to give any desired separation of the critical pole and zero. There are an infinite number of values of K_t and K_a which will give a particular separation of P_1 and Z_1, however, a minimum ω_n will normally be desired, and this value should be set equal to Z_1, to determine the necessary K_a. It is then simple to determine the necessary K_t from the relations below.

$$Z_1 = [K/(KK_a + a_2)]^{\frac{1}{2}}$$

$$(\omega_n)^2_{min} = K/(KK_a + a_2)$$

$$K_a = 1/(\omega_n)^2_{min} - a_2/K$$

$$P_1 = Z_1 + \text{desired separation}$$

$$= (KK_t + a_1)^{\frac{1}{2}}$$

$$K_t = (P_1^2 - a_1)/K$$

The constants a_1 and a_2 are, respectively, the coefficients of the first and second order terms of the uncompensated characteristic equation.

Figure 4 shows the movement of the critical pole and zero for varying acceleration and tachometer feedback gains, and the effect of varying system gain. Intersections of the curves indicate that the critical pole and zero lie one on top of the other. These points are the limits of stability for the system. Therefore, the areas between the curves, and to the right of an intersection, are stable regions for the system. For a given KK_a, the frequency of the critical zero can be determined. Likewise, for a given KK_t, the frequency of the critical

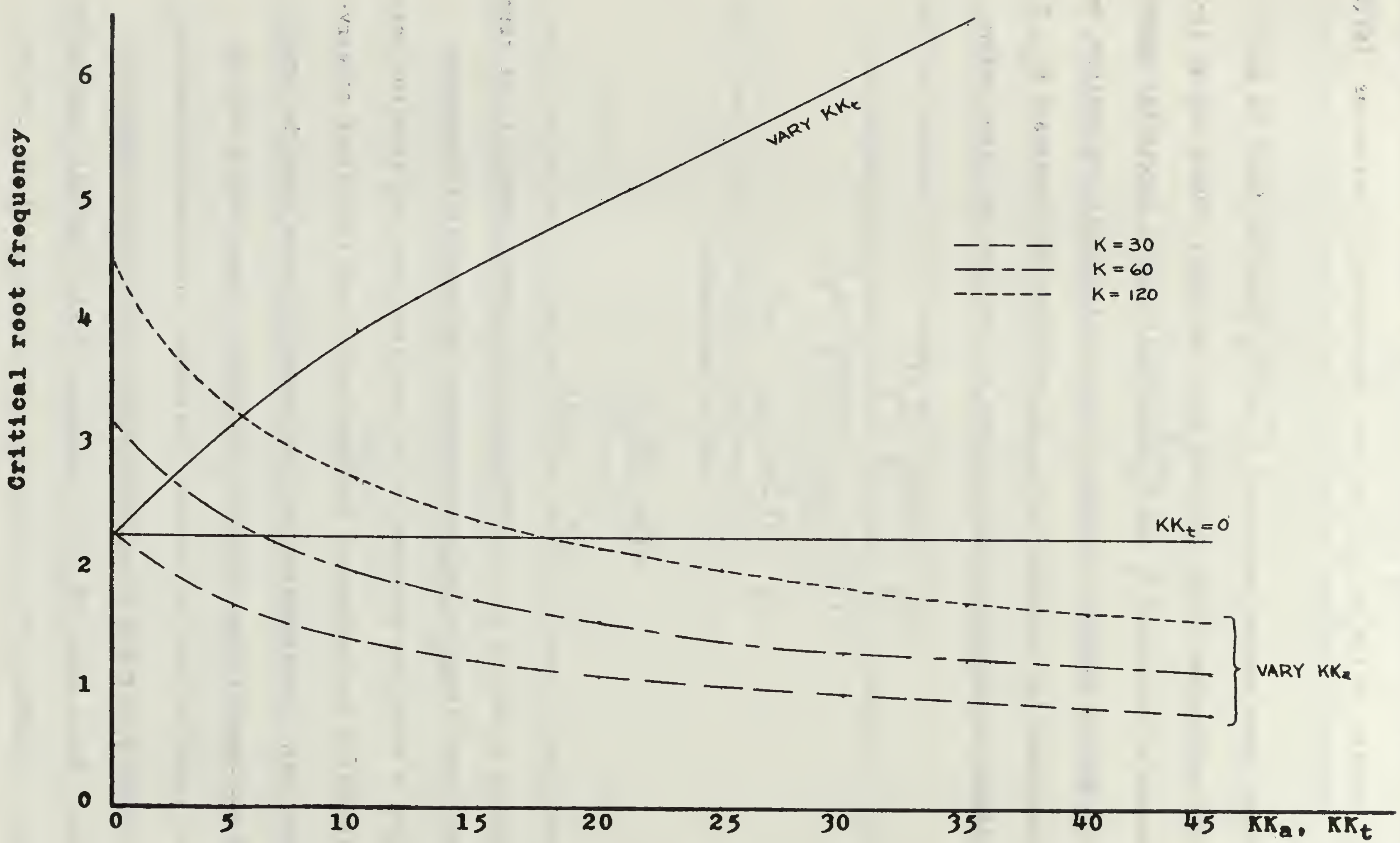


Fig. 4. Plot of critical pole-zero locations for varying tachometer and acceleration feedback gains, and various system gains.

pole can be found. The vertical separation between the two points found on the curves, is the separation between the critical pole and zero on the imaginary axis of the S-plane. The separation can thus be picked off the curves for any desired values of tachometer and acceleration feedback gains.

Figure 5 shows the effect of varying system gain and tachometer feedback gain, with no acceleration feedback. The only intersection shown on Figure 5 is with the system gain curve and the $KK_t = 0$ curve. It is obvious, however, that for sufficiently high values of K and for low values of KK_t, the zero on the K curve would be above the pole of the KK_t curve, and instability would result. Therefore, the comments concerning Figure 4 also apply to Figure 5. The intersection on Figure 5 shows that the uncompensated system will be unstable for system gains greater than 30. The addition of sufficient tachometer feedback, however, will give any desired separation between P_1 and Z_1.

As an example of the use of the curves, assume $K = 24$, $K_t = 2.0$, and $K_a = 0$. From Figure 5, Z_1 would be at a frequency of 2.0 radians per second and P_1 at a frequency of 7.3 radians per second. From this information, approximate values for the system damping factor and natural frequency are:

$$\omega_n = 2.0 + \tfrac{1}{2}(7.3 - 2.0)$$

$$= 4.65 \text{ radians/second}$$

$$\zeta = 2.65/\left[(2.65)^2 + (4.65)^2\right]^{\frac{1}{2}}$$

$$= 0.495$$

Figure 6 is the root locus plot for this sample system. From this plot, it is seen that the actual values are

$$\omega_n = 6.53 \text{ radians/second}$$

$$\zeta = 0.383.$$

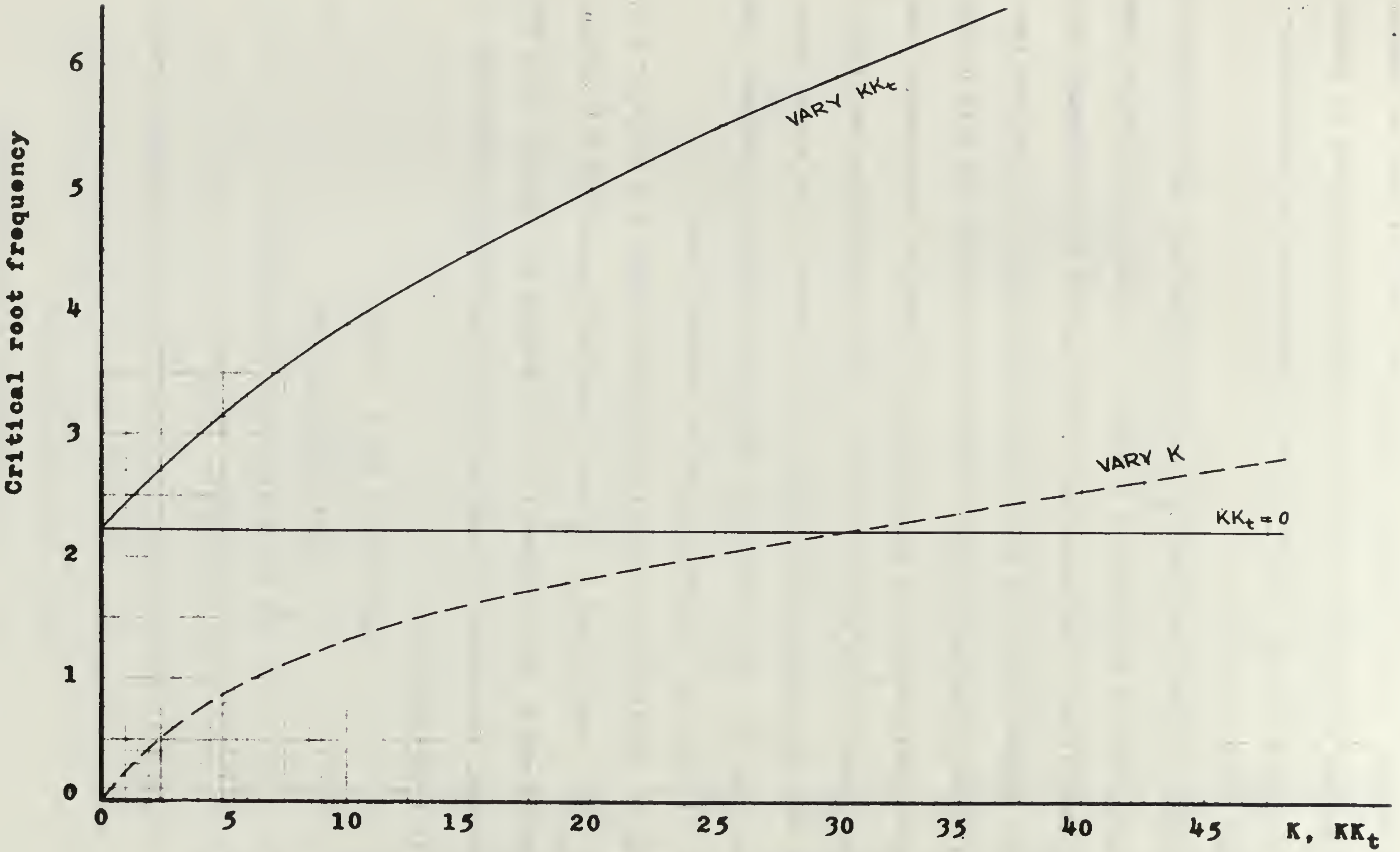


Fig. 5. Critical root locations of 3^{rd} order system with varying system and tachometer feedback gains.

Obviously, the approximations to the actual values are very rough. The reason for the differences in the values is that the approximation assumed the root would lie on the midpoint of a semi-circular lobe, while the root was actually closer to the pole, and the lobe was not semi-circular. This difference will be discussed later, in the design of compensation for example systems.

Figures 7, 8, 9, and 10 are root locus plots for the third order system with different combinations of tachometer and acceleration feedback gains. The location of the roots, on the root loci lobes, are shown, and demonstrate the effects discussed earlier.

It was seen from the even and odd parts of the closed-loop characteristic equation, that the even part is a function of two parameters, K and KK_a, and the odd part is a function of one parameter, KK_t. By setting the parameter of the odd part equal to a constant, it should be possible to plot stability curves with the ordinate and abscissa variables the two parameters of the even part. Figure 11 is a family of such curves. They show the limit of stability for constant KK_t while varying K and KK_a. The system is stable in the region below the curves, and is unstable above the curves. These curves demonstrate the same stability information as the curves of Figures 4 and 5, but in a different manner.

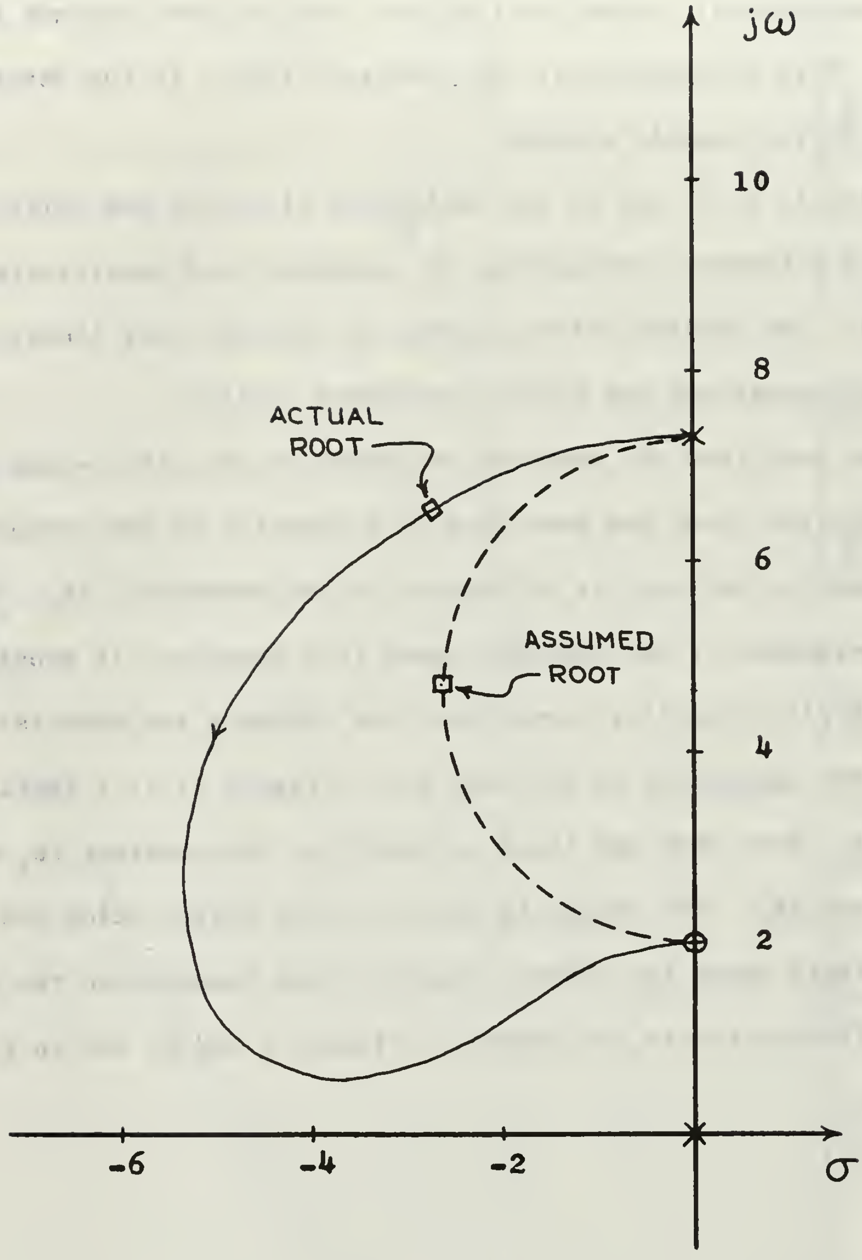


Fig. 6. Root locus of 3^{rd} order system with $K_t = 2.0$ and $K_a = 24.0$.

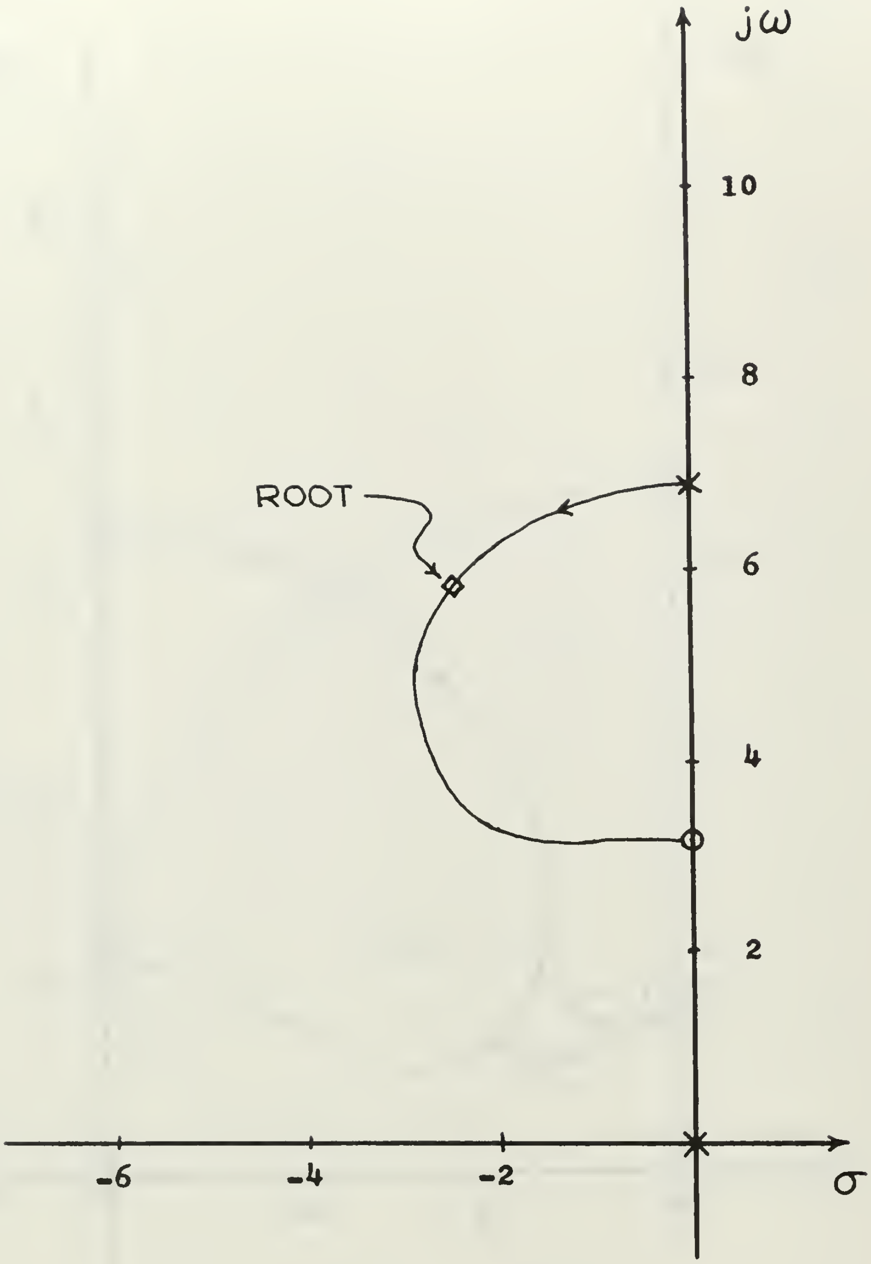


Fig. 7. Root locus of 3^{rd} order system with $K_t = 0.7$.

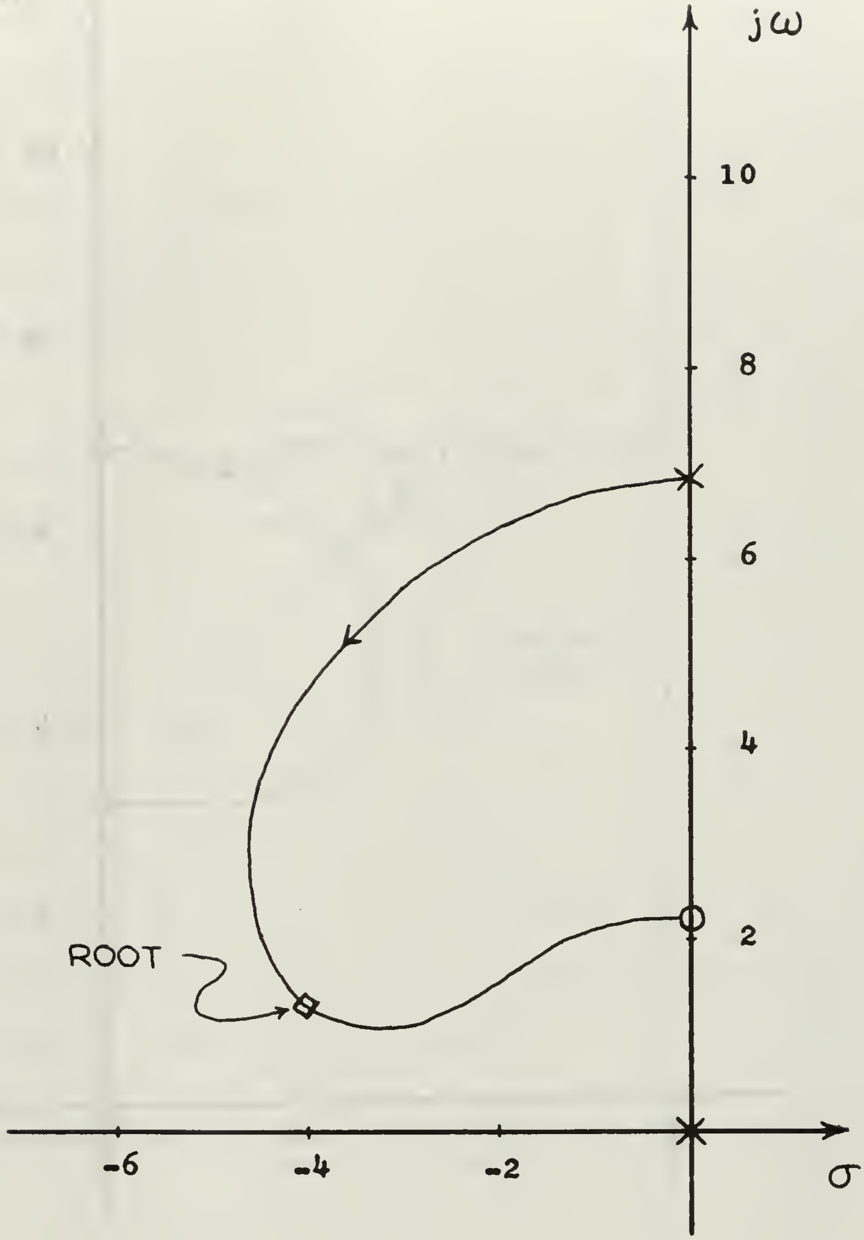


Fig. 8. Root locus of 3^{rd} order system with $K_t = 0.7$ and $K_a = 0.1$.

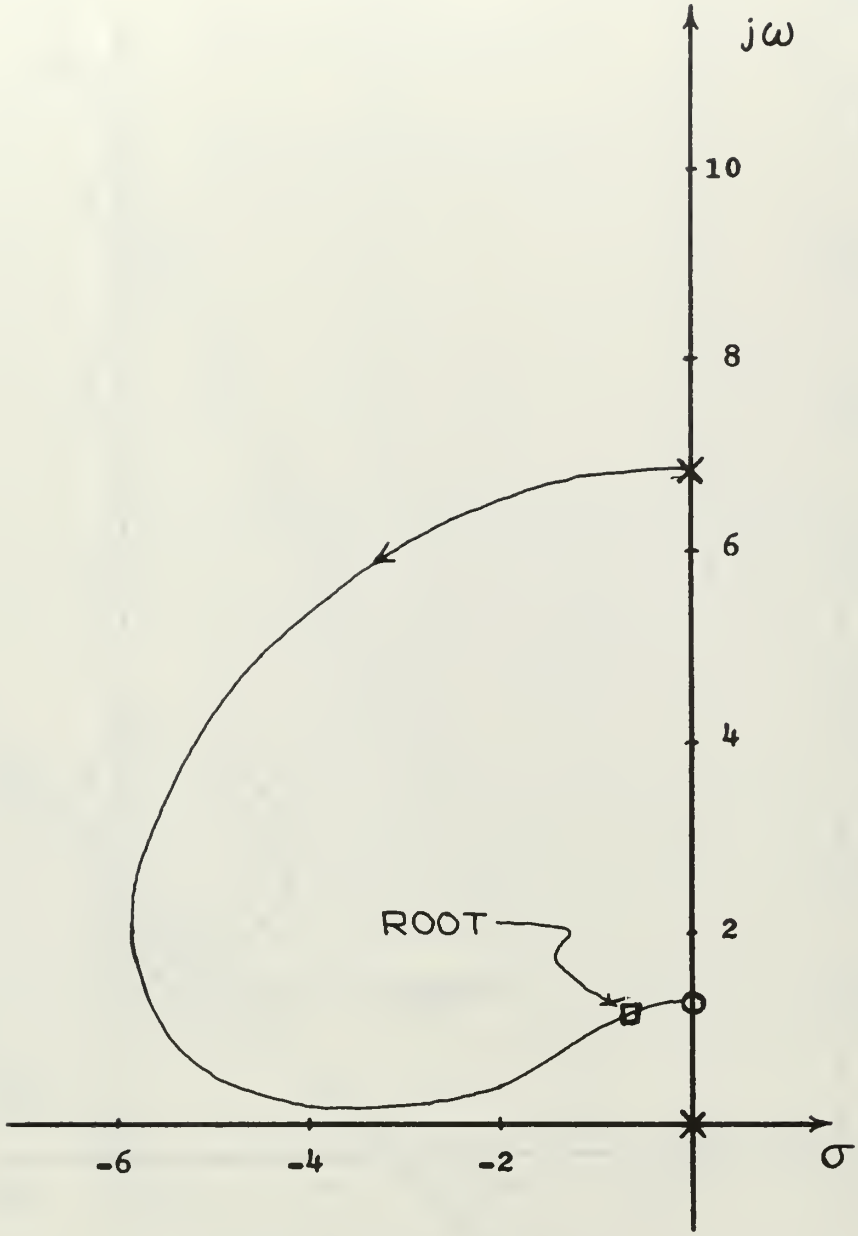


Fig. 9. Root locus of 3^{rd} order system with $K_t = 0.7$ and $K_a = 0.5$.

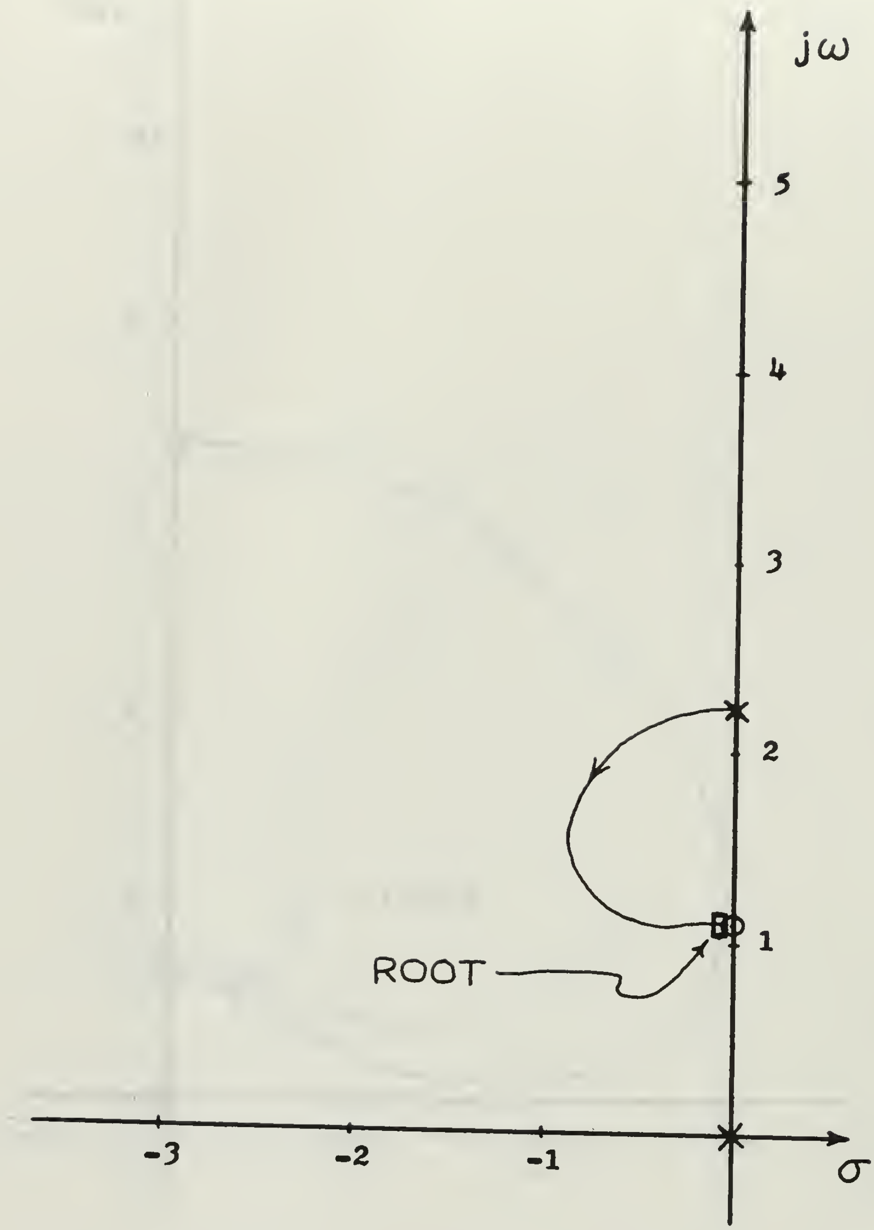


Fig. 10. Root locus of 3^{rd} order system with $K_t = 0$ and $K_a = 0.7$.

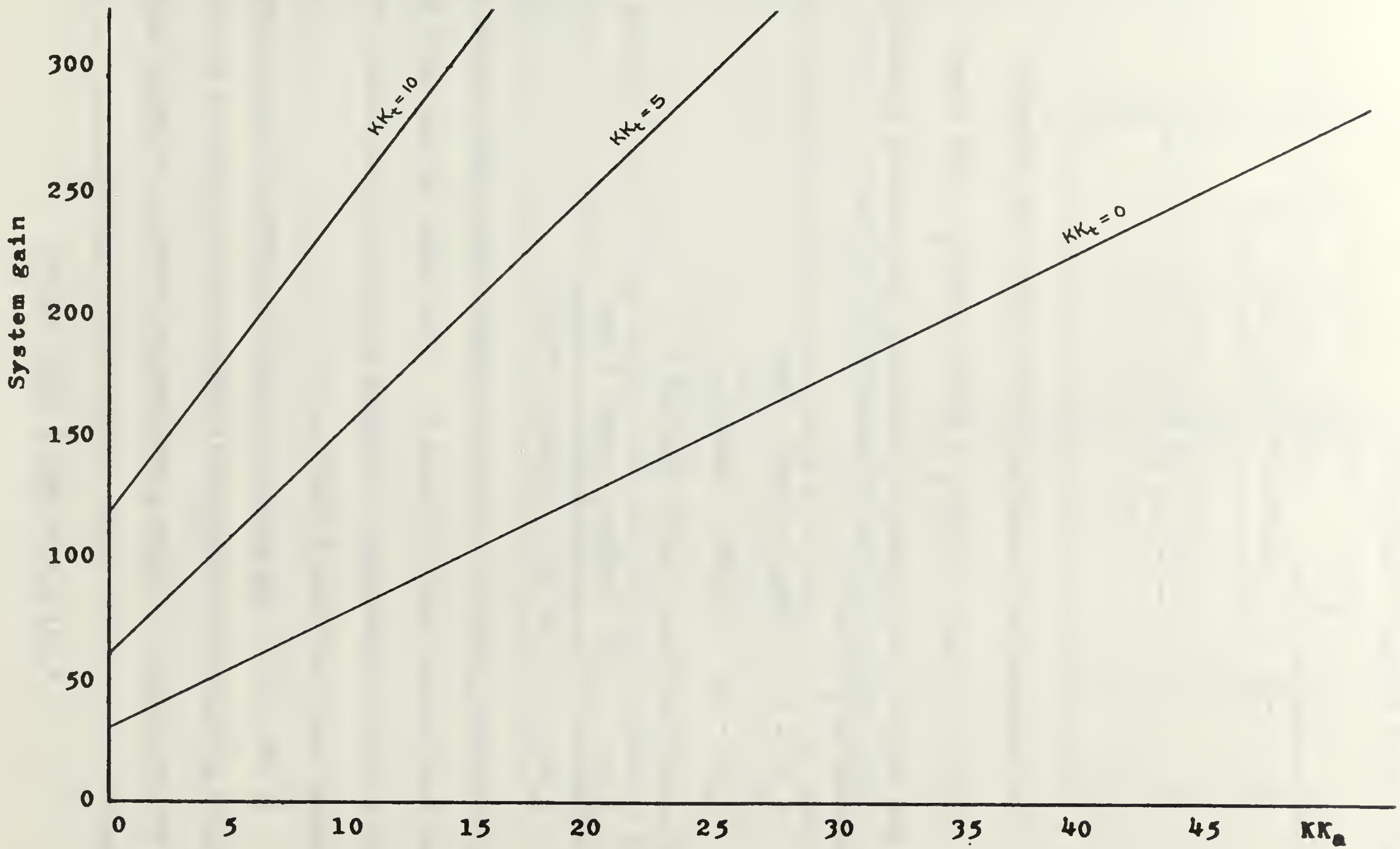
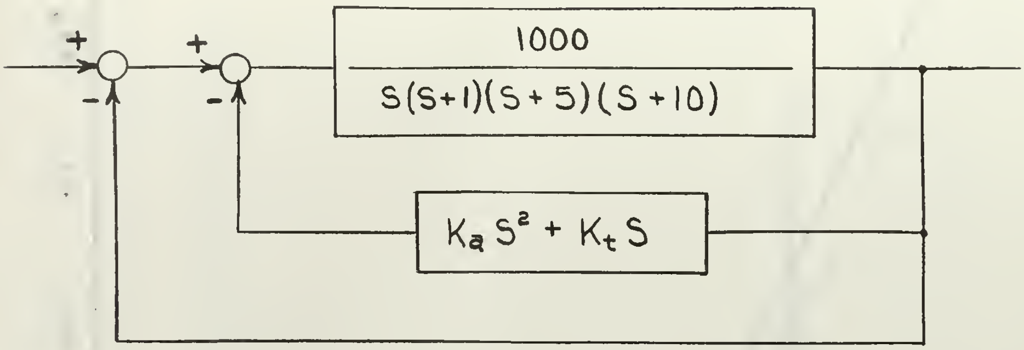


Fig. 11. Stability curves of 3rd order system.

3. Stability analysis of fourth order system with tachometer and acceleration feedback compensation.



The characteristic equation of this closed-loop system is

$$F(S) = S^4 + 16S^3 + (1000K_a + 65)S^2 + (1000K_t + 50)S + 1000.$$

To use the root locus stability criterion, this equation is partitioned into even and odd parts

$$F_e(S) = S^4 + (1000K_a + 65)S^2 + 1000$$

$$F_o(S) = 16S^3 + (1000K_t + 50)S.$$

The proper root locus form is obtained as

$$\frac{F_e(S)}{F_o(S)} = \frac{S^4 + (1000K_a + 65)S^2 + 1000}{16S\,[S^2 + (1000K_t + 50)/16]} = -1.$$

Setting K_a and K_t = 0 yields the basic system, which has the pole-zero distribution shown in Figure 12. This system is seen to be unstable without compensation. In order to stabilize the system, it is necessary to interchange P_1 and Z_1.

As was seen in the analysis of the third order system, the addition of acceleration feedback will effect only the zeros of the system. To see the effects of varying acceleration feedback and system gains, consider the even part of the characteristic equation.

$$F_e(S) = S^4 + (KK_a + 65)S^2 + K = 0$$

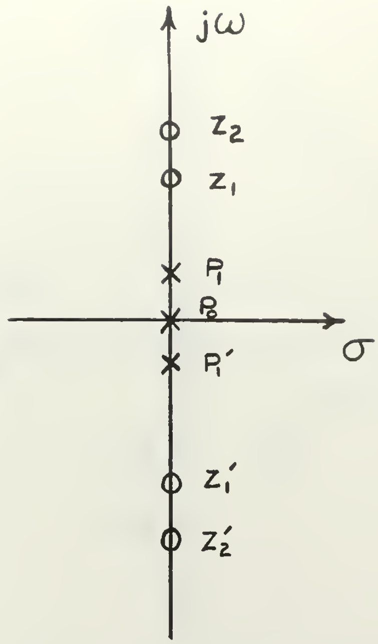


Fig. 12. Pole-zero distribution of basic 4^{th} order system.

Partitioning this equation and putting it in root locus form yields

$$\frac{KK_a S^2}{S^4 + 65S^2 + K} = -1.$$

In general, the root locus of this form of $F_e(S)$, will appear as shown in Figure 13. It is seen that the addition of acceleration feedback will move Z_1 in the proper direction to stabilize the system.

With no tachometer feedback, the poles will remain fixed, as seen from $F_o(S)$ with $K_t = 0$.

$$F_o(S) = 16S(S^2 + 3.12)$$

Therefore, it is always possible to stabilize a fourth order system with the addition of sufficient acceleration feedback.

Since the addition of tachometer feedback will effect only the poles of the system, a study of $F_o(S)$ will determine the movements of P_1 while varying tachometer feedback and system gains.

$$F_o(S) = 16S^3 + (KK_t + 50)S = 0$$

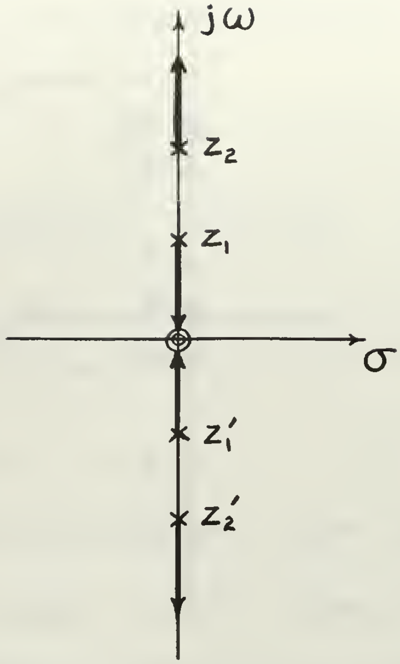


Fig. 13. Root locus of $F_e(S)$ for increasing acceleration feedback gain.

Partitioning this equation and putting it in root locus form yields

$$\frac{KK_t}{16(S^2 + 3.12)} = -1.$$

The root locus of this form of $F_o(S)$ will appear as shown in Figure 14(a). With no acceleration feedback, the root locus form of $F_e(S)$ becomes

$$\frac{K}{S^2(S^2 + 65)} = -1.$$

This root locus is shown in Figure 14(b).

The addition of tachometer feedback is seen to move P_1 in the proper direction for stability. Figure 14(b), however, shows that for sufficiently large values of system gain, the zeros of the system become complex. In the general case, $F_e(S)$ will be of the form

$$F_e(S) = S^4 + bS^2 + K.$$

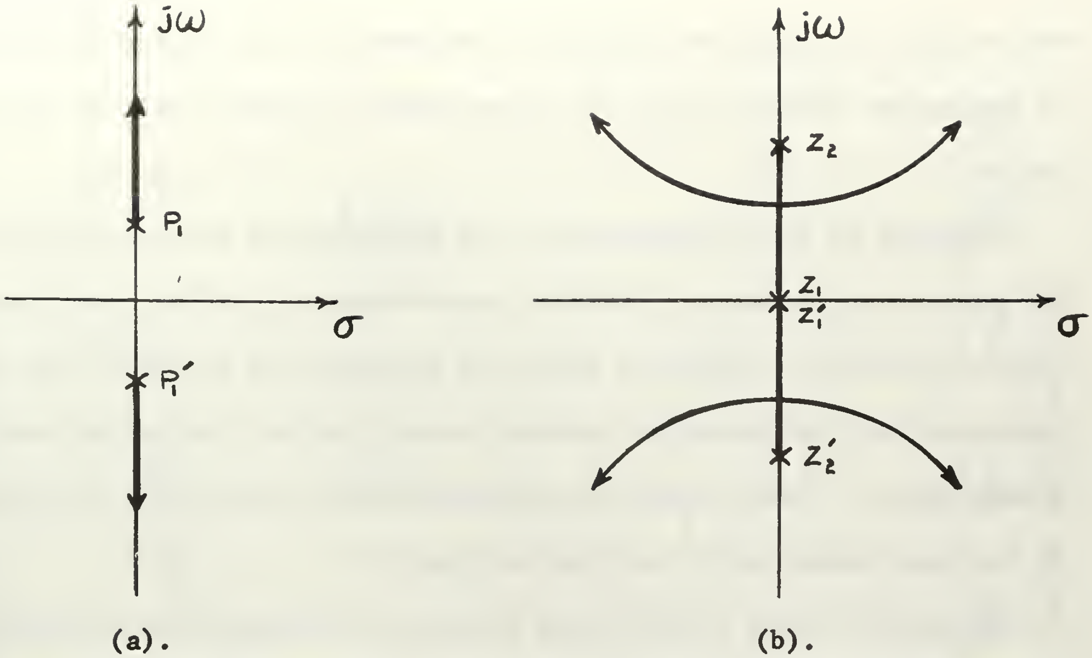


Fig. 14. (a). Root locus of $F_o(S)$ for increasing tachometer feedback gain. (b). Root locus of $F_e(S)$ for increasing system gain, $K_a = 0$.

The zeros will become complex when

$$K > b^2/4.$$

The root locus stability criterion states that all poles and zeros must lie on the imaginary axis, and in the proper sequence, for stability. Therefore, a system with complex zeros is unstable. As the addition of tachometer feedback does not effect the zeros of the system, when $K > b^2/4$ the system can not be stabilized by tachometer feedback alone. In this case, acceleration feedback is required to move the zeros back to the imaginary axis in order to achieve stability.

For a fixed value of system gain, less than required to make the zeros complex, Figures 14(a) and (b) show that the addition of tachometer feedback will stabilize the system. If the tachometer feedback gain is increased sufficiently, however, P_1 will move above Z_2 and instability will result. Therefore, tachometer feedback alone may or may

not be able to stabilize a fourth order system, and large enough values of tachometer feedback gain will always make a fourth order system unstable.

Figures 15 and 16 demonstrate the movements of the system poles and zeros when tachometer feedback, acceleration feedback, and system gains are varied. Figure 15 shows the movements of P_1 and Z_1 for varying tachometer and acceleration feedback gains, and for various values of system gains. These curves were generated from root locus calculations, in the same manner as the curves of Figure 4.

Points on these curves could also have been generated directly, although with more difficulty than was the case for the third order system. The location of P_1 varies as $(KK_t + 50)^{\frac{1}{2}}/4$, and Z_1 varies as $\left\{KK_a + 65 - [(KK_a + 65)^2 - 4K]^{\frac{1}{2}}\right\}^{\frac{1}{2}}$. The curves in Figure 15 are very similar to those of Figure 4, for the third order system, and the same comments apply.

Figure 16 shows the movements of P_1, Z_1 and Z_2 as system and tachometer feedback gains are varied, without acceleration feedback. For this system, it is seen that Z_1 and Z_2 become complex for $K \approx 1056$, and the system can not be stabilized with tachometer feedback alone for system gains greater than this value. It is also seen that P_1 moves above Z_2 for sufficiently high tachometer feedback gains, causing instability.

From the curves in Figures 15 and 16, certain conclusions may be drawn concerning the effects of various compensation schemes. It has already been stated that tachometer feedback alone may or may not be capable of stabilizing the system, and that acceleration feedback alone can always stabilize a fourth order system. Figure 15 shows that a proper combination of acceleration and tachometer feedback will give any desired separation between the critical poles and zeros. As with

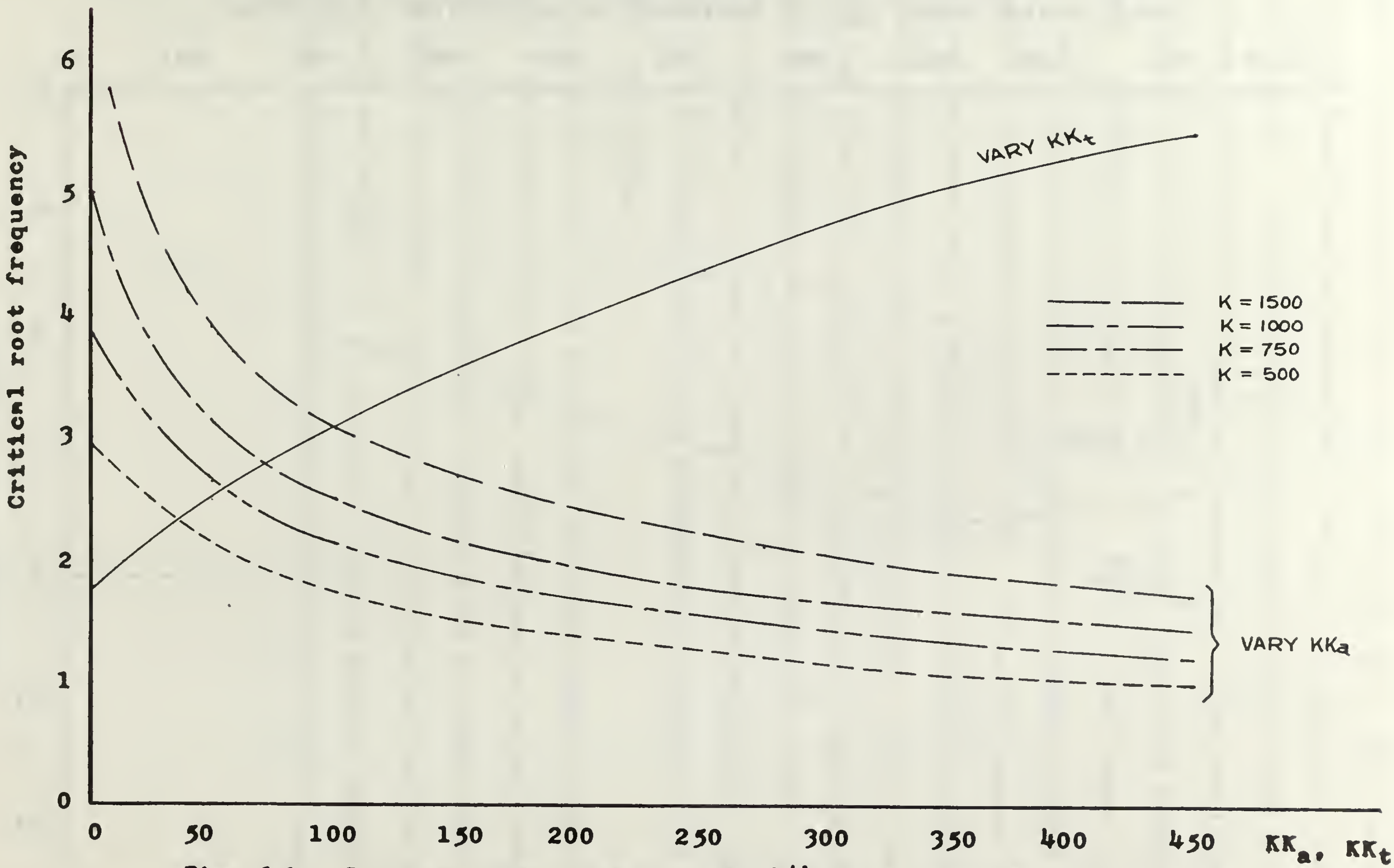


Fig. 15. Critical root locations of 4th order system with varying system, tachometer feedback and acceleration feedback gains.

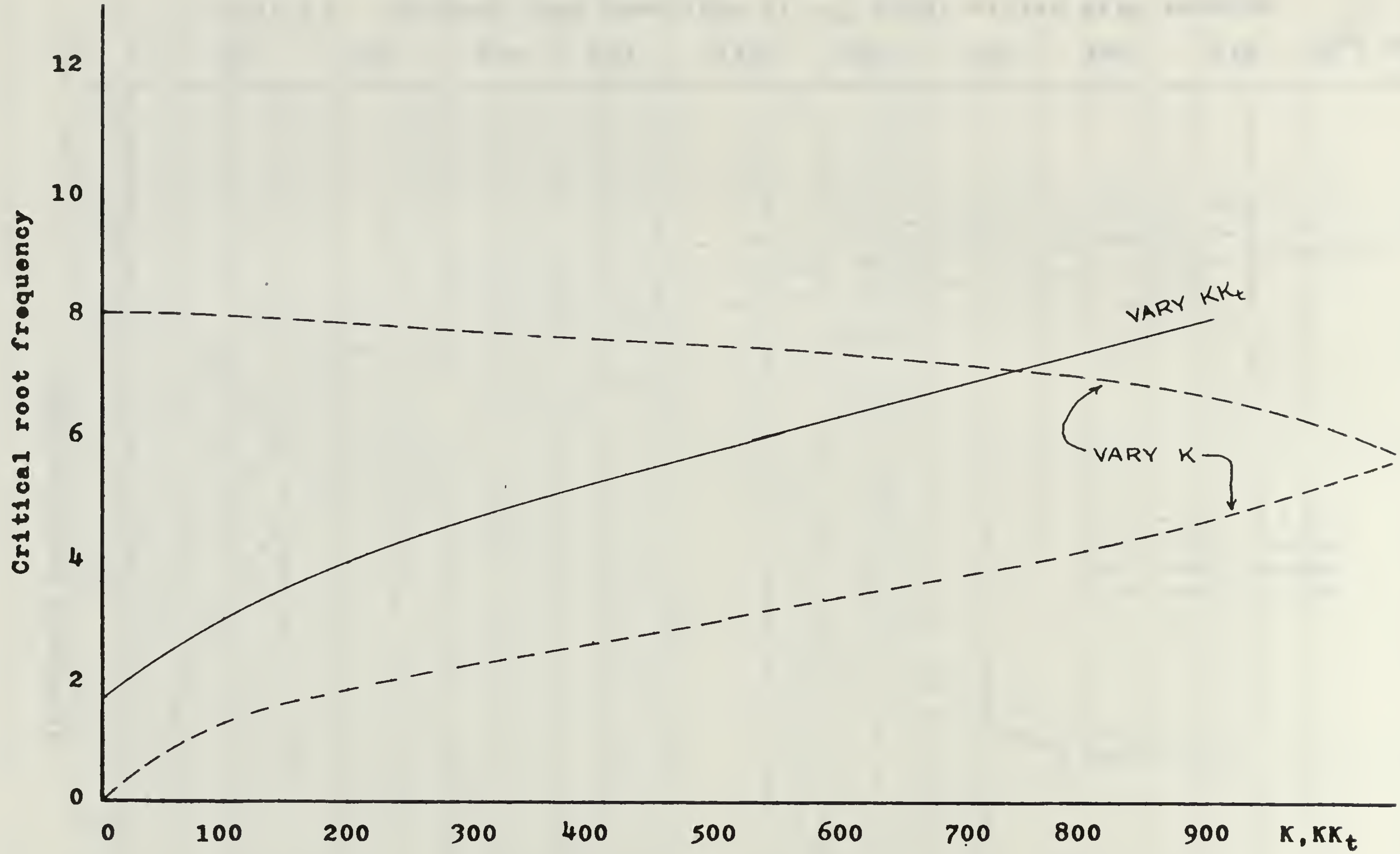


Fig. 16. Critical root locations of 4th order system with varying system and tachometer feedback gains.

the third order system, the damping and natural frequency of the system depend upon the location of the roots on the root loci. The location of the roots depend upon the separation of the poles and zeros, and upon the root locus gain. Unlike the third order system, the root locus gain does not vary with changes in acceleration feedback gain. The root locus gain is the inverse of the coefficient of the S^3 term, which remains constant. Therefore, it is expected that ζ and ω_n will depend entirely upon the separation between the system poles and zeros, and upon their location on the imaginary axis of the S-plane. It would therefore be expected that even though tachometer feedback may stabilize the system, it would not normally produce satisfactory performance. With a constant root locus gain, it might also be expected that it would be rather difficult to determine the required amount of separation, between the poles and zeros, to locate the roots for a desired performance. This point will be discussed further when compensation is designed for a fourth order system.

Figures 17, 18, 19, and 20 are root loci of the example fourth order system. The system roots are shown, and the effects of various combinations of tachometer and acceleration feedback gains are demonstrated. It is seen, with the small root locus gain of this system, that quite large pole-zero separations are required to achieve adequate damping. The large separations cause the system to have a high natural frequency.

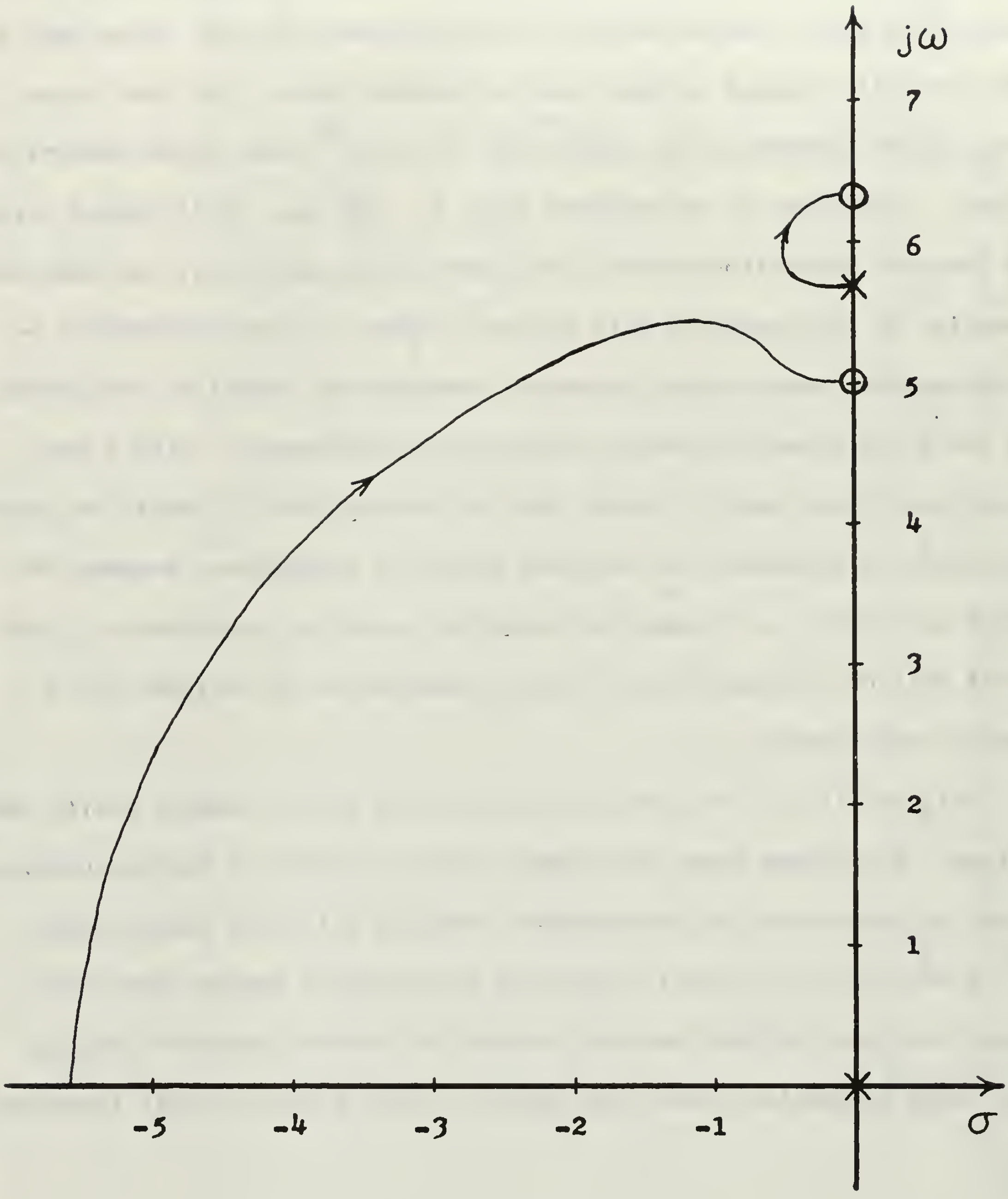


Fig. 17. Root locus of 4^{th} order system with $K_t = 0.47$ and $K_a = 0$.

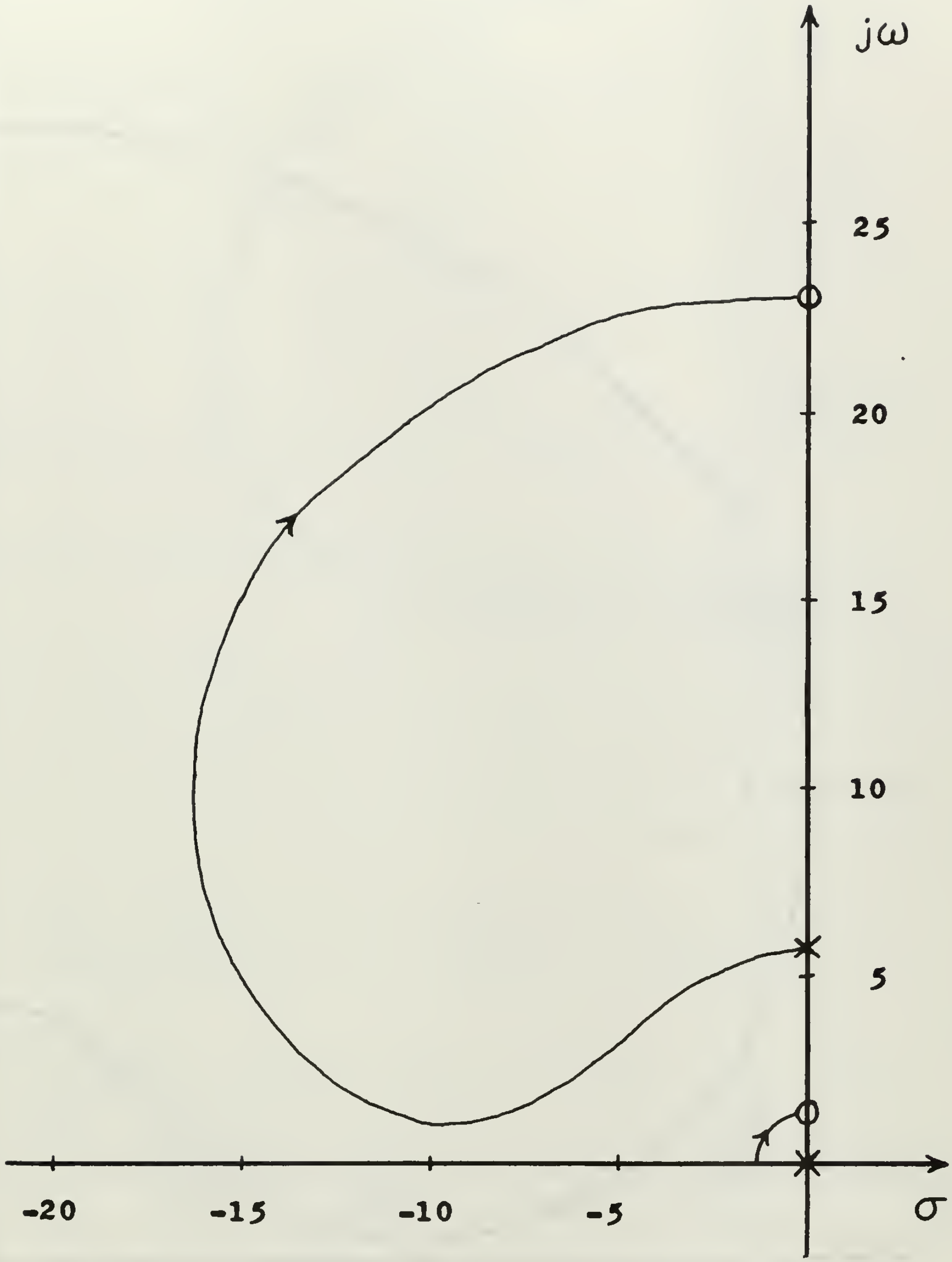


Fig. 18. Root locus of 4th order system with K_t = 0.47 and K_a = 0.47.

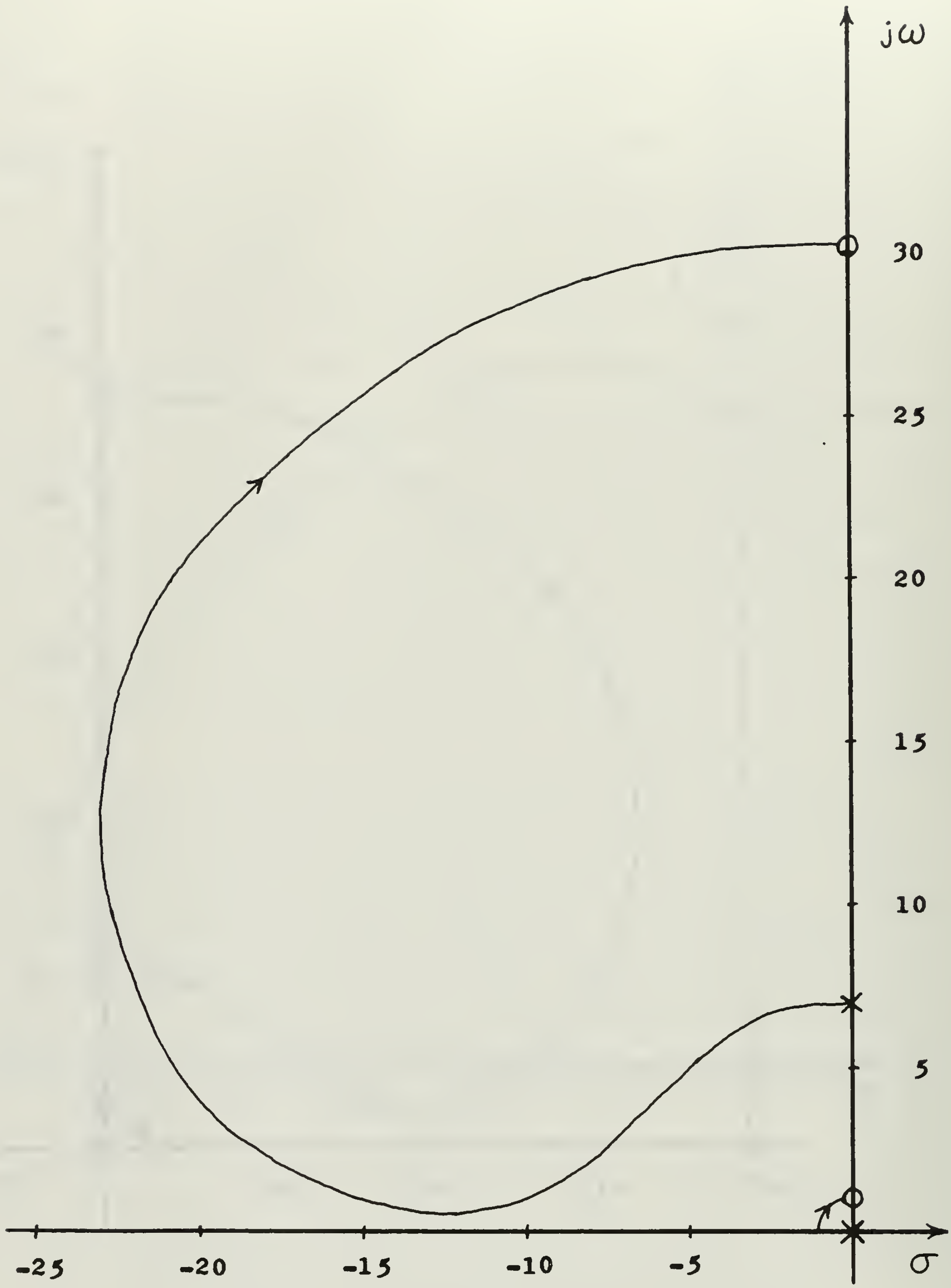


Fig. 19. Root locus of 4^{th} order system with $K_t = 0.74$ and $K_a = 0.85$.

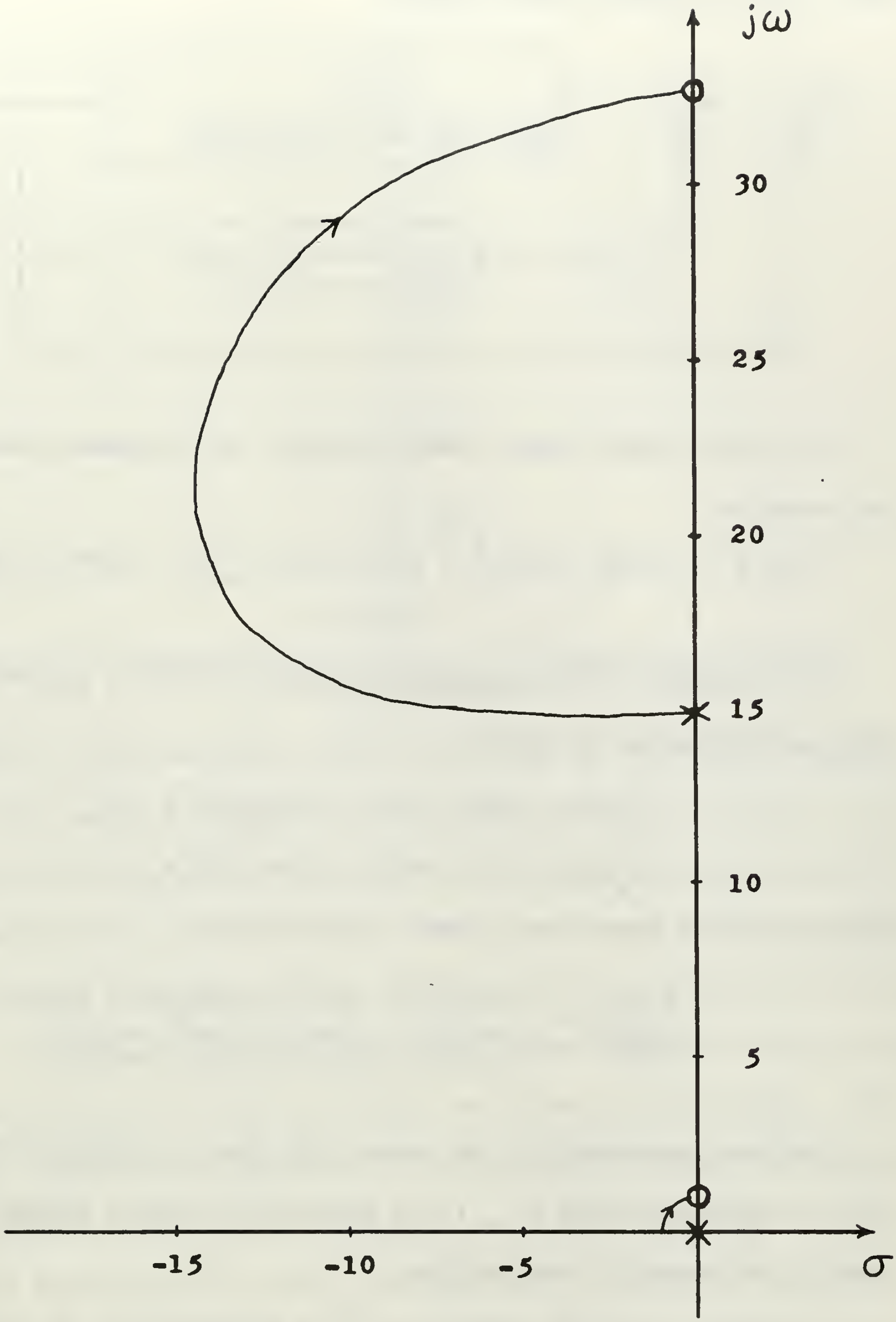
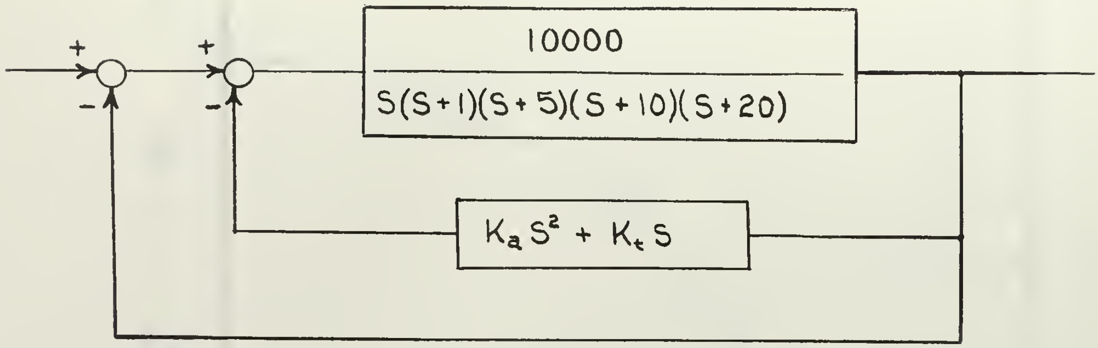


Fig. 20. Root locus of 4^{th} order system with $K_t = 3.5$ and $K_a = 1.0$.

4. Stability Analysis of a Fifth Order System with Tachometer and Acceleration Feedback Compensation.



The fifth order system shown above has the following characteristic equation

$$F(S) = S^5 + 36S^4 + 385S^3 + (1350 + 10^4 K_a)S^2 + (1000 + 10^4 K_t)S + 10^4 = 0.$$

For analysis, the characteristic equation will be partitioned into even and odd parts, as before.

$$F_e(S) = 36S^4 + (1350 + 10^4 K_a)S^2 + 10^4$$

$$F_o(S) = S^5 + 385S^3 + (1000 + 10^4 K_t)S$$

The proper root locus form is then obtained as

$$\frac{F_e(S)}{F_o(S)} = \frac{36 [S^4 + (37.5 + 278K_a)S^2 + 278]}{S [S^4 + 385 S^2 + (1000 + 10^4 K_t)]} .$$

Without compensation, the system will have a pole-zero distribution as shown in Figure 21. It is seen that the basic system is unstable, and requires compensation.

In order to achieve stability, it is necessary to interchange the positions of P_1 and Z_1, without disturbing the relative locations of P_2 and Z_2. The locations of Z_1 and Z_2 are governed by $F_e(S)$, which is a quadratic in S^2. A study of $F_e(S)$ reveals that as acceleration feedback

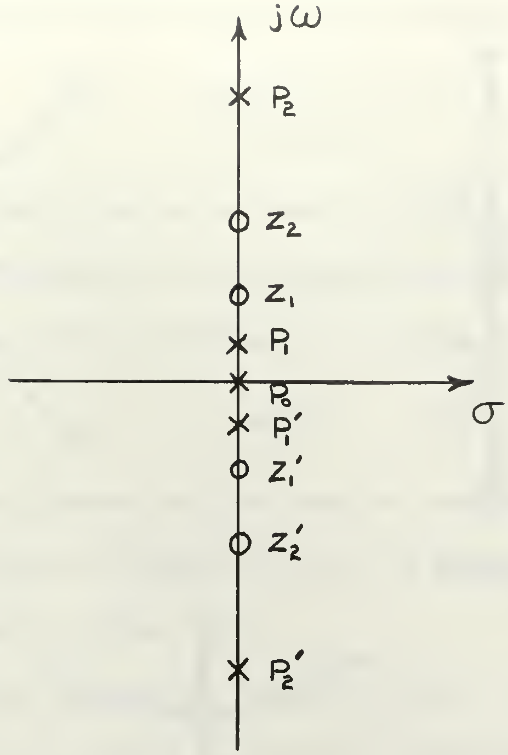


Fig. 21. Pole-zero distribution of 5^{th} order system without compensation.

is added, Z_1 will move down, and Z_2 will move up the S-plane imaginary axis. The locus of zeros, for increasing acceleration feedback, is shown in Figure 22(a). Therefore, it may be possible to stabilize the system with acceleration feedback alone; however, too large a value of acceleration feedback gain will cause Z_2 to move above P_2, resulting in instability. It should also be noted that large values of system gain will cause the roots of $F_e(S)$ to become complex, as shown in Figure 22(b). To achieve stability in this case, it is necessary to return the roots to the imaginary axis by means of adding acceleration feedback.

The locations of P_1 and P_2 are governed by the quadratic, in S^2, of $F_o(S)$. A study of this quadratic shows that the poles move closer together on the imaginary axis, as tachometer feedback is added. The locus of the poles, for increasing tachometer feedback gain, is shown

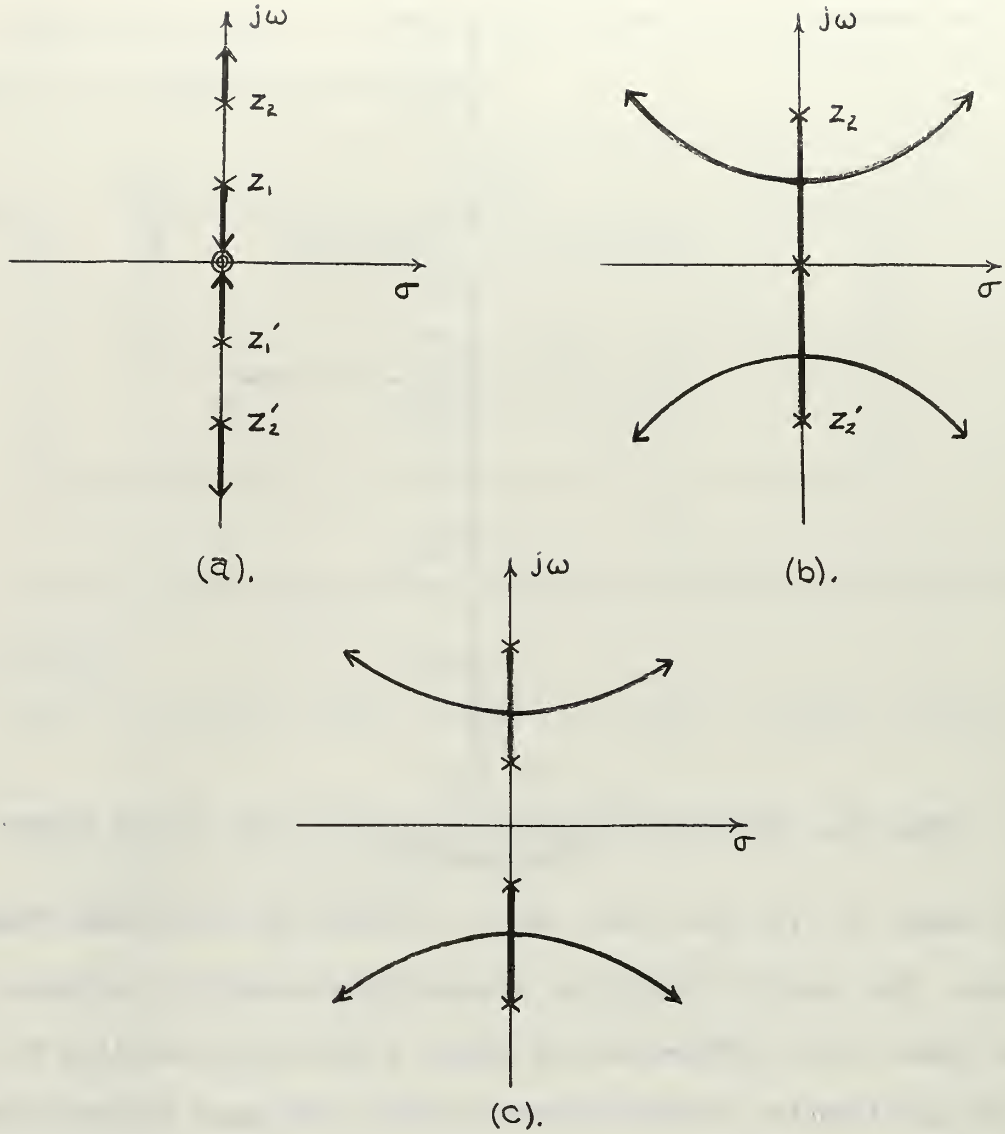


Fig. 22. (a). Root locus of $F_e(S)$ for increasing K_a. (b). Root locus of $F_e(S)$ for increasing K, $K_a = 0$. (c). Root locus of $F_o(S)$ for increasing K_t.

in Figure 22(c). Tachometer feedback alone may be capable of stabilizing the system; however, it is seen that large values of tachometer feedback gain cause the poles to become complex. Therefore, there is an absolute maximum value of tachometer feedback which may be used without causing instability. In general,

$$F_o(S) = S\ [S^4 + a_3S^2 + (a_1 + KK_t)].$$

The maximum tachometer feedback gain which may be used is

$$K_t = \frac{(a_3/2)^2 - a_1}{K}$$

The above conclusion can be extended to define the condition under which a fifth order system may not be stabilized with tachometer and/or acceleration feedback. This condition is that for

$$a_1 > (a_3/2)^2$$

the poles of the basic system will be complex, and compensation by tachometer and/or acceleration feedback will not be capable of returning the poles to the imaginary, S-plane axis.

The pole and zero movements, discussed above, are shown in Figures 23 and 24. The curves in these figures were generated from root locus calculations for the even and odd parts of F(S). Figure 23 is the locus of pole-zero movements when tachometer feedback and system gains are varied, and acceleration feedback is zero. The curves demonstrate that there are maximum values of the system and tachometer feedback gains, which may be used without the system poles becoming complex. For this system, the values are

$$K = 12600$$

$$K_t = 3.61.$$

The curves also show that tachometer feedback alone is capable of stabilizing a fifth order system, when the system zeros are not complex.

Figure 24 shows the pole-zero movements for varying tachometer and acceleration feedback gains, and for various values of system gain. These curves indicate that the proper values of tachometer and acceleration feedback should always be capable of stabilizing a fifth order system, providing the system gain is not too high. As before, it is noted

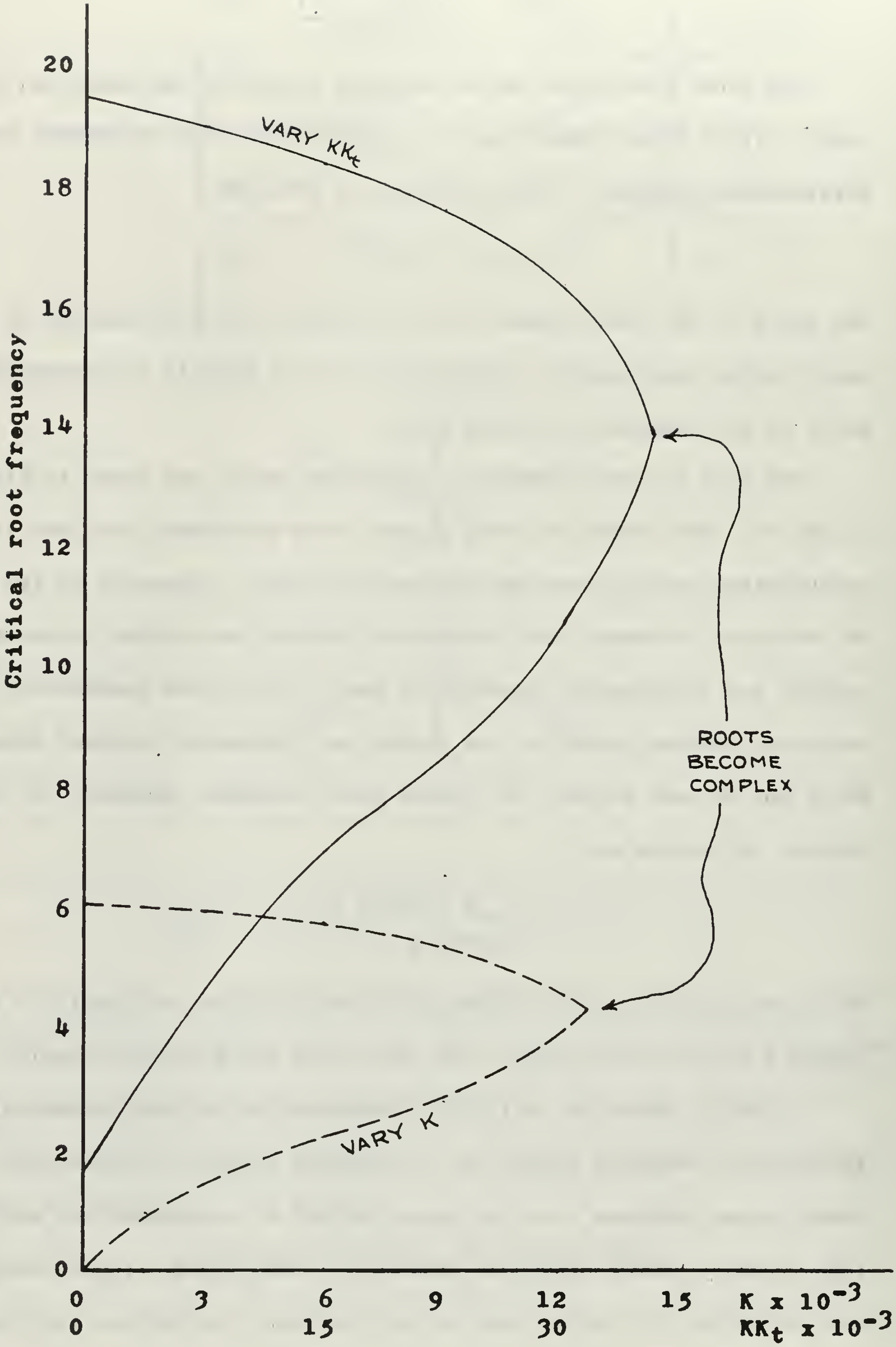


Fig. 23. Critical root locations of 5^{th} order system.

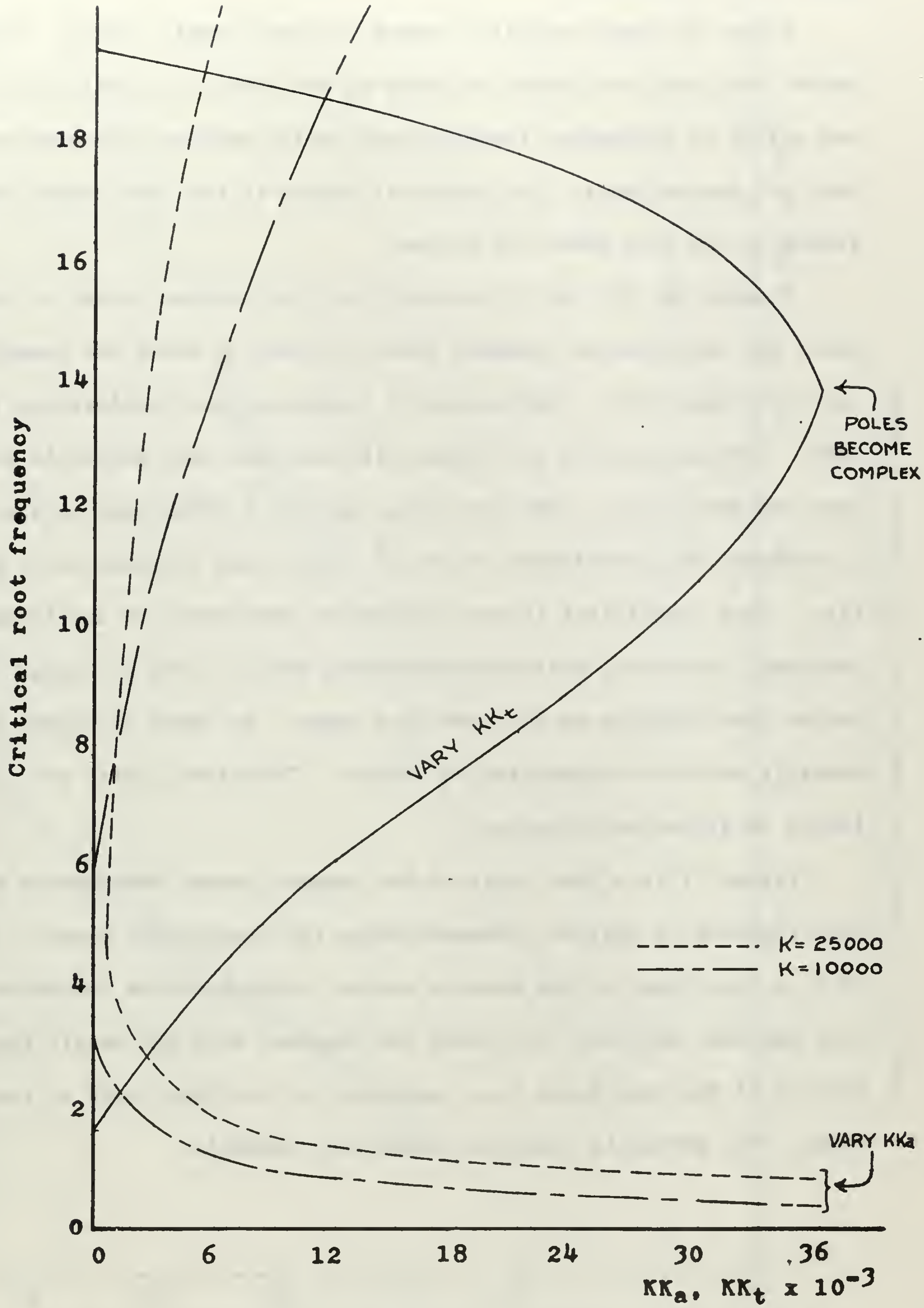


Fig. 24. Critical root locations of 5^{th} order system with varying tachometer and acceleration feedback gains.

that large values of tachometer and acceleration feedback gains will always cause instability.

Figure 25 shows stability curves for the example system. These curves are part of a family of plots of the limit of stability for various values of tachometer feedback gain, while varying acceleration feedback and system gains. The system is stable in the area below, and unstable in the area above the curves.

Figures 26, 27, and 28 are root loci for various values of tachometer and acceleration feedback gains. Figure 26 shows the example system stabilized with a combination of tachometer and acceleration feedback. Although stable, the system will not have very desirable performance characteristics. The root locus gain of a fifth order system, is a constant, the coefficient of the S^5 term in the characteristic equation. This coefficient is not affected by tachometer or acceleration feedback; therefore, pole-zero separation must be used to change the system root location on the root loci lobes. As shown in Figure 24, the possible pole-zero separation is limited. Therefore, there are also limits on system performance.

Figure 27 is a root locus of the example system when excess acceleration feedback is applied, demonstrating the instability caused. Figure 28 is a root locus of the example system to which excess tachometer feedback has been applied. The poles are complex, with the result that one portion of the root locus lies completely in the right half of the S-plane. The system is therefore absolutely unstable.

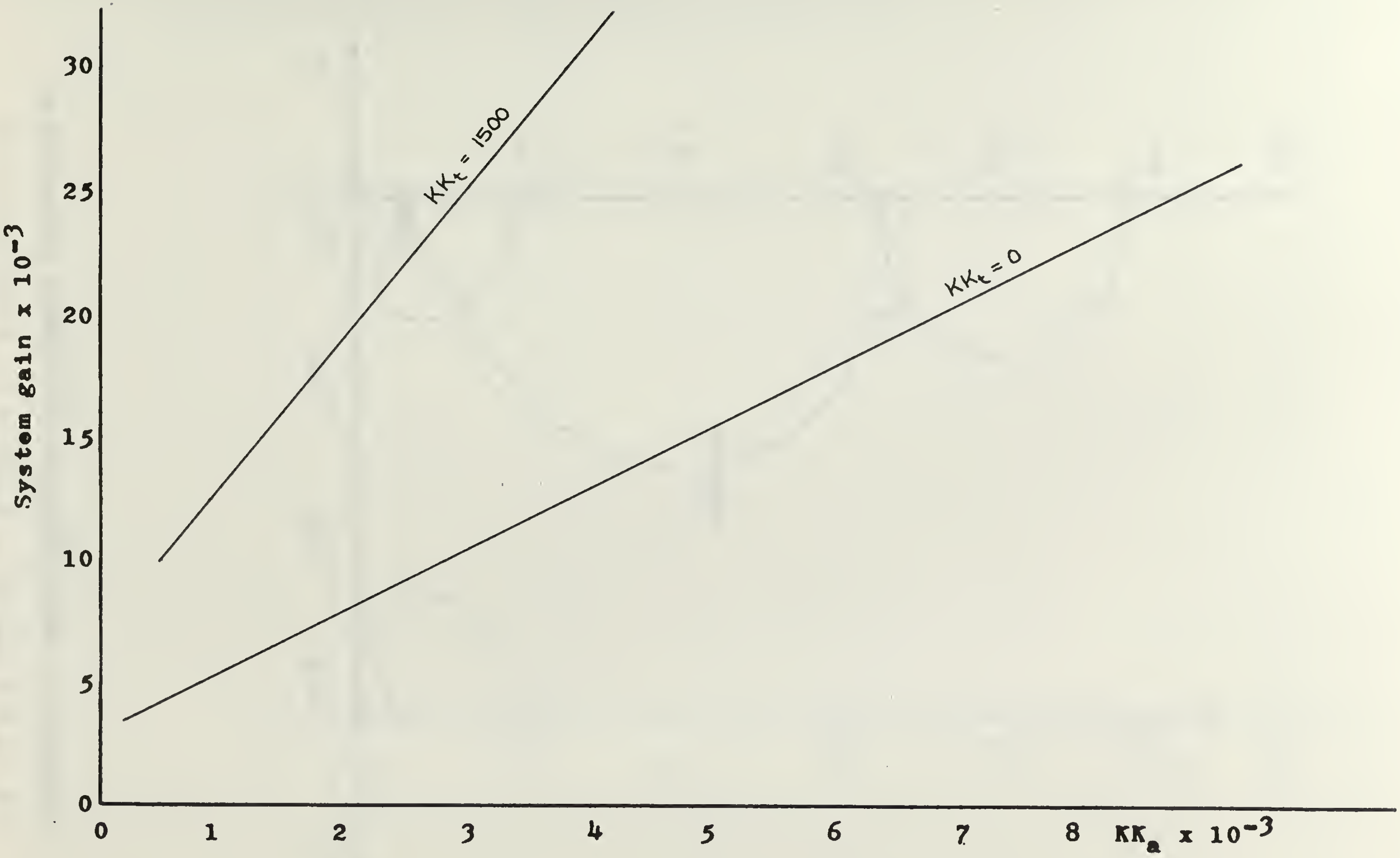


Fig. 25. Stability curves of 5th order system.

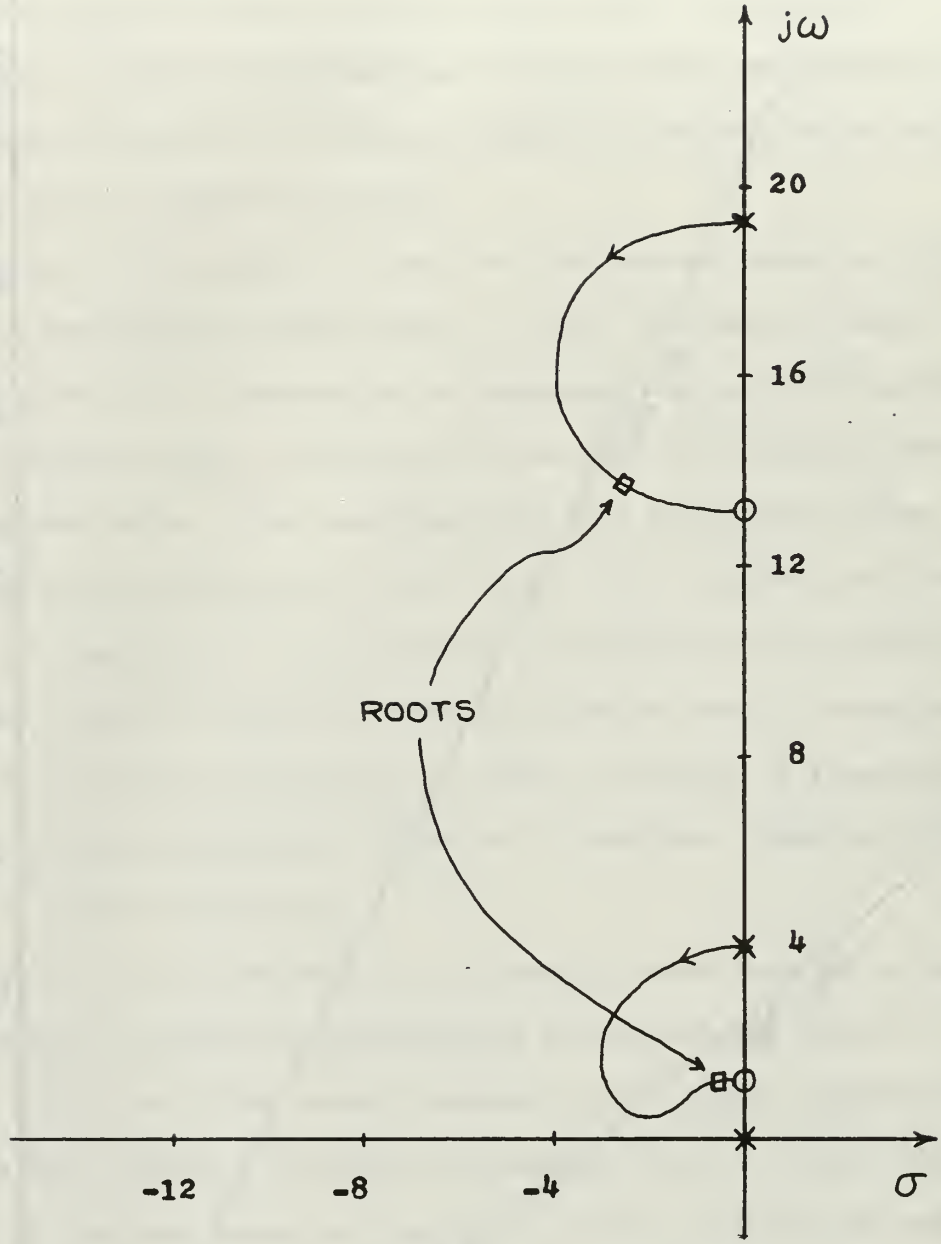


Fig. 26. Root locus of 5th order system with K_t = 0.5 and K_a = 0.5.

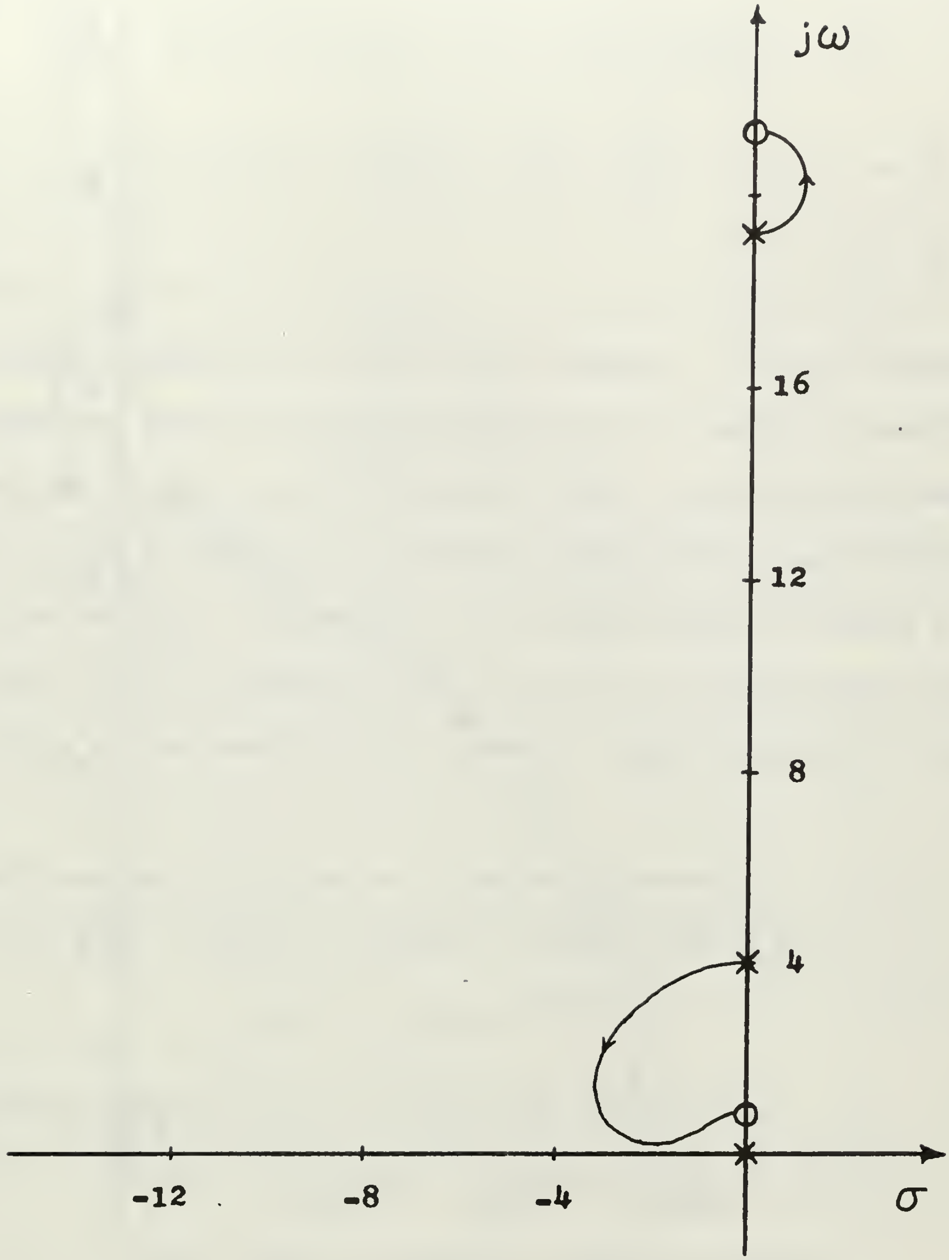


Fig. 27. Root locus of 5^{th} order system with $K_t = 0.5$ and $K_a = 1.5$.

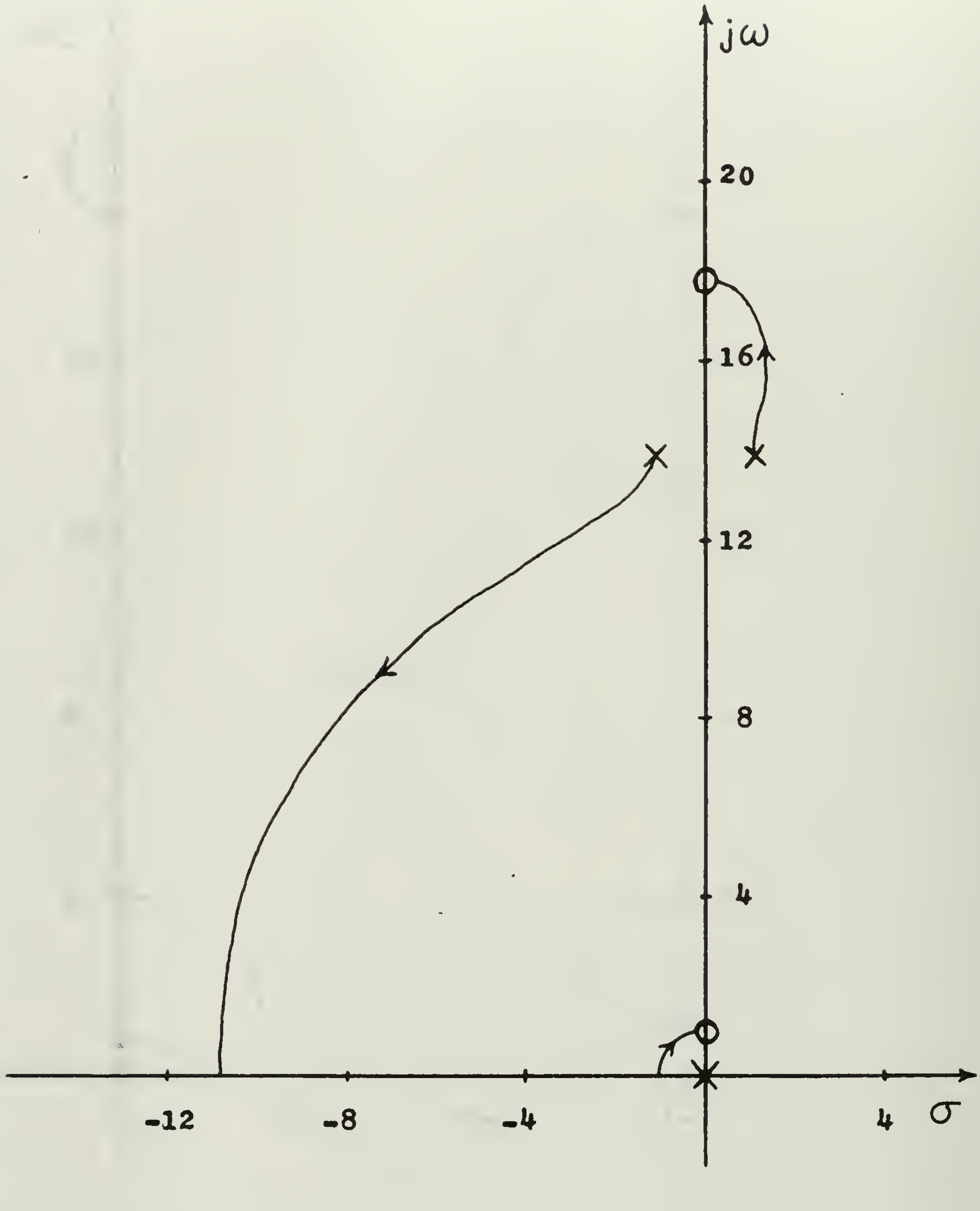
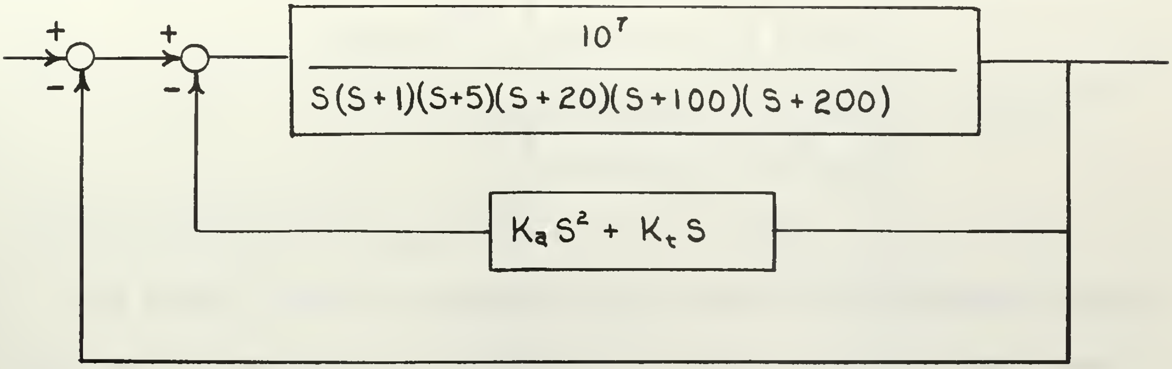


Fig. 28. Root locus of 5th order system with $K_t = 3.7$ and $K_a = 1.0$.

5. Stability analysis of a sixth order system with tachometer and acceleration feedback compensation.



From the results of the fifth order analysis, it is not expected that tachometer and acceleration feedback will be capable of providing good response from a sixth order system. Under certain circumstances, however, it may be possible to stabilize a sixth order system with this type of compensation. An analysis of the above system, by means of the root locus stability criterion, will point out the possibilities and difficulties of compensation with tachometer and/or acceleration feedback.

The characteristic equation of the above system is

$$F(S) = S^6 + 326S^5 + 2245S^4 + 403520S^3 + (401600 + 10^7 K_a)S^2$$
$$+ (2X10^6 + 10^7 K_t)S + 10^7 = 0$$

This equation is partitioned into even and odd parts as

$$F_e(S) = S^6 + 2245S^4 + (401600 + 10^7 K_a)S^2 + 10^7$$

$$F_o(S) = 326S^5 + 403520S^3 + (2X10^6 + 10^7 K_t)S .$$

The required root locus form is

$$\frac{F_e(S)}{F_o(S)} = \frac{S^6 + 2245S^4 + (401600 + 10^7 K_a)S^2 + 10^7}{326S(S^4 + 1237.8S^2 + 6134.97 + 35674.8K_t)} = -1.$$

With $K_a = K_t = 0$, the basic system will have a pole zero distribution as shown in Figure 28(a).

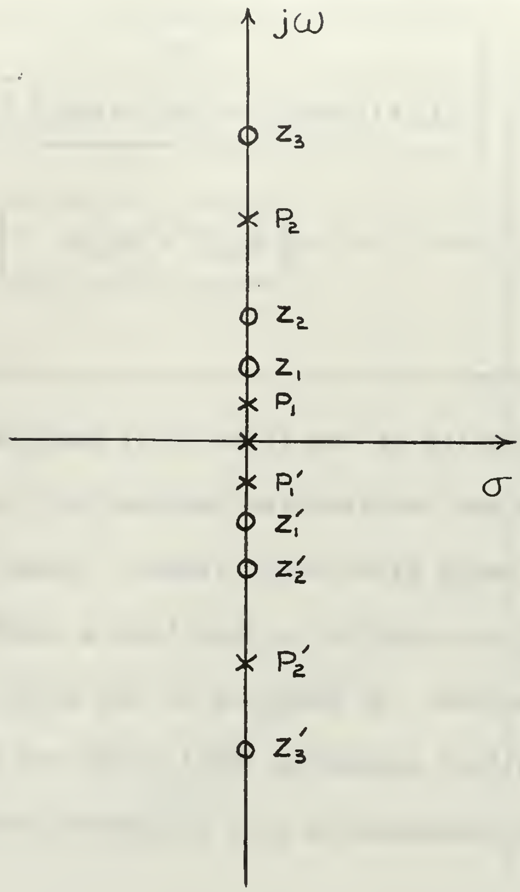


Fig. 28(a). Pole-zero distribution of basic 6^{th} order system.

The system is unstable. In order to stabilize it, the locations of P_1 and Z_1 must be interchanged.

The locations of the system poles are governed by the quadratic in S^2, contained in $F_o(S)$. Figure 29(a) shows the locus of pole movements for increasing tachometer feedback gain. It is seen that adding tachometer feedback moves P_1 and P_2 closer together on the imaginary axis. Therefore, it may be possible to stabilize the system with tachometer feedback alone. As with the fifth order system, there is an absolute maximum value of tachometer feedback gain which may be used without causing the poles to become complex, thereby causing instability. In

general, this value is

$$K_t = \frac{(a_3/2a_5)^2 - a_1/a_5}{K}$$

where: a_1 = coefficient of S term

a_3 = coefficient of S^3 term

a_5 = coefficient of S^5 term.

For this system, the maximum value is $K_t = 0.0377$.

As before, there is also a condition under which tachometer and/or acceleration feedback are not capable of stabilizing the sixth order system. This condition is

$$a_1/a_5 > (a_3/2a_5)^2$$

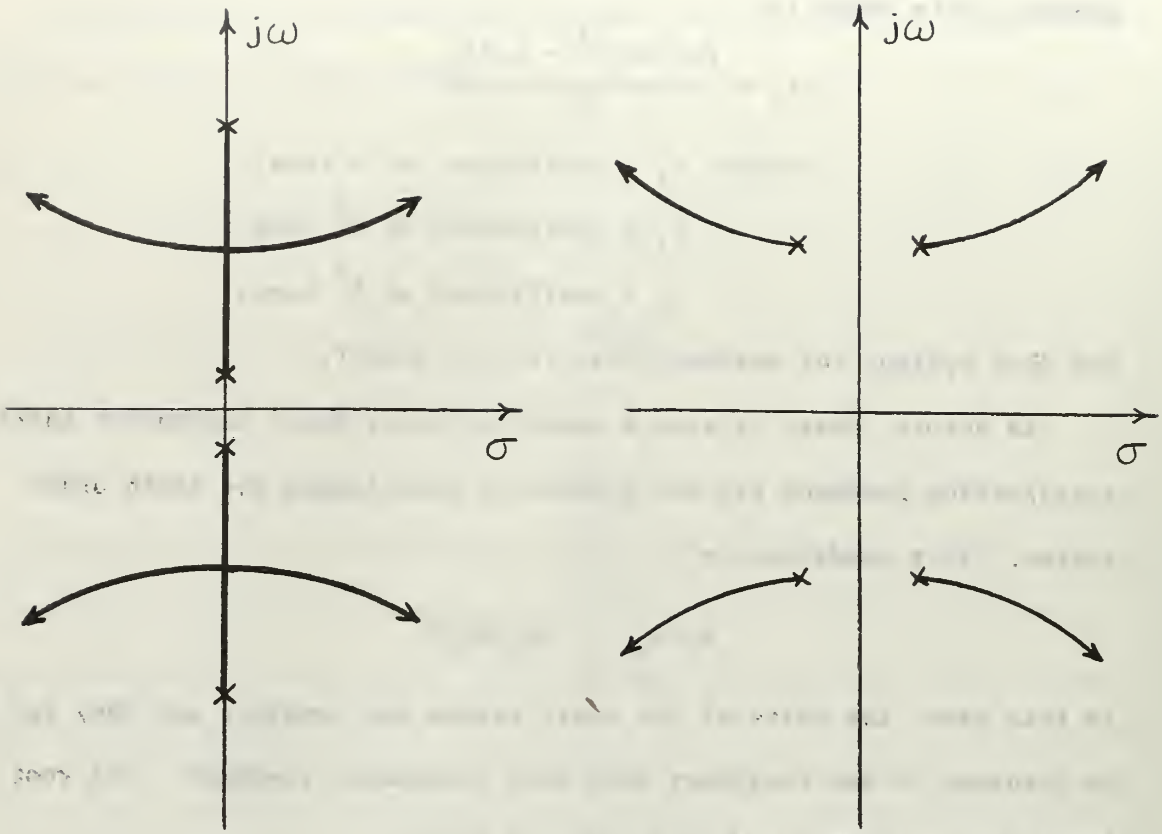
In this case, the poles of the basic system are complex, and they can not be returned to the imaginary axis with tachometer feedback. The root locus for this case is shown in Figure 29 (b).

The system zero locations are governed by $F_e(S)$, a cubic in S^2. The roots of the cubic may all be negative real, yielding six zeros on the imaginary axis, or the roots may consist of one negative real root and one pair of complex conjugate roots. In this case, there would be two roots on the imaginary axis, and four complex roots located symmetrically about the origin of the S-plane.

The example system has all six zeros on the imaginary axis. To study the effect of adding acceleration feedback, $F_e(S)$ is partitioned and put in root locus form.

$$\frac{10^7 K_a S^2}{S^6 + 2245S^4 + 401600S^2 + 10^7} = -1$$

Figure 30 (a) shows the locus of zero movements as acceleration feedback gain is increased. It is seen that Z_1 moves in the proper direction for

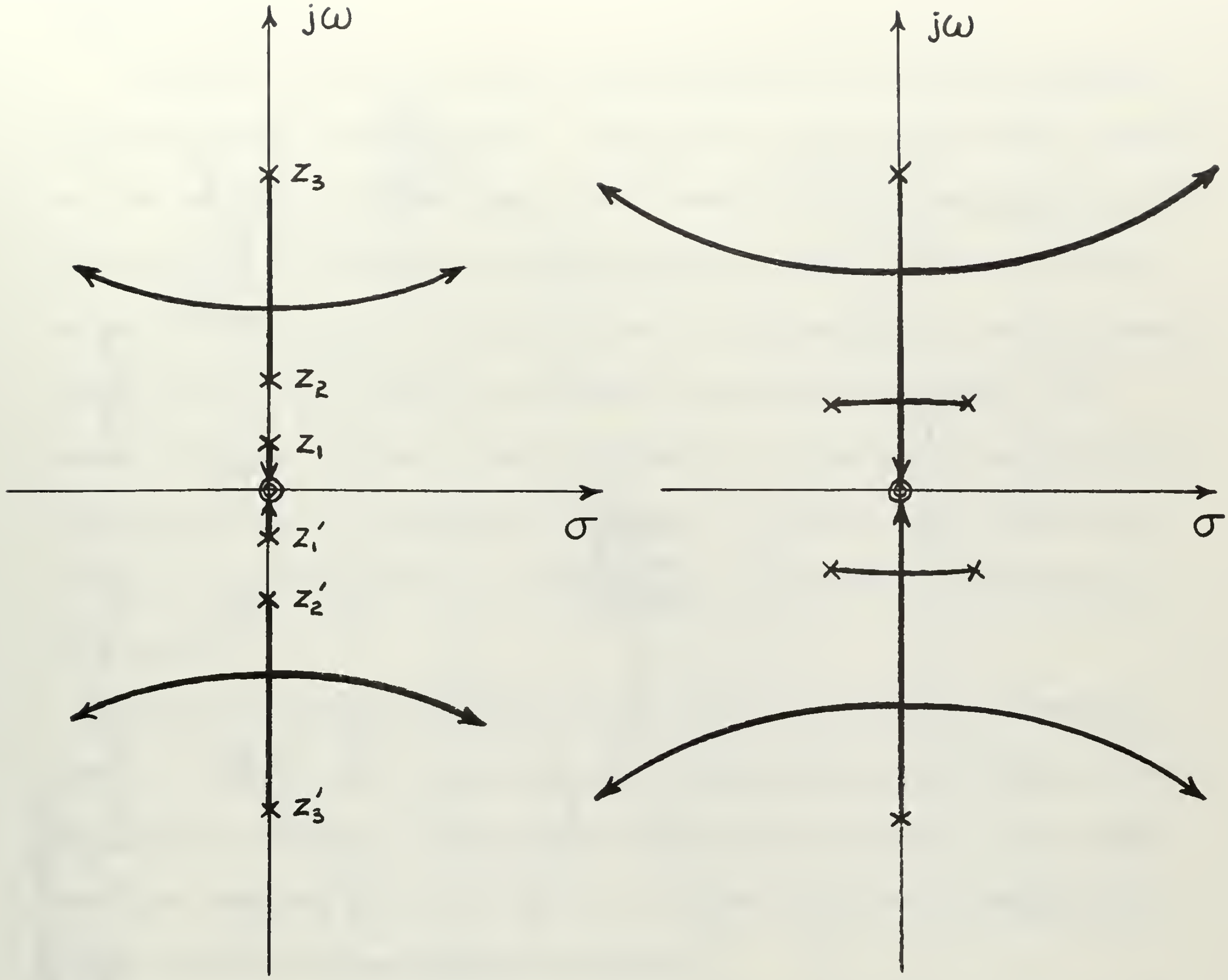


(a). (b).

Fig. 29. (a). Locus of pole movements with increasing K_t.
(b). Locus of pole movements with increasing K_t, poles initially complex.

stability. Z_2 moves up the axis, however, and there is the possibility that it will move above P_2 before the system can be stabilized. There is also a maximum value of acceleration feedback gain, which may be used without causing Z_2 and Z_3 to become complex. This value is not easily found algebraically; however, for a given system it may be found very simply from third order Mitrovic curves. [2] It may also be found from root locus calculations. For the example system, this maximum value is $K_a = 0.087$.

Figure 30 (b) shows the locus of zero movements for increasing acceleration feedback gain, when Z_2 and Z_3 are initially complex. It is seen that acceleration feedback will return the zeros to the imaginary



(a). (b).

Fig. 30. (a). Locus of zero movements, with increasing K_a, for 6th order system with all zeros on the imaginary axis. (b). Locus of zero movements, with increasing K_a, for 6th order system with two complex conjugate pairs of zeros.

axis. The comments concerning Figure 30(a) also apply, however, and acceleration feedback alone may or may not be capable of stabilizing a sixth order system.

Figure 31 is a plot of pole-zero movements when acceleration feedback is zero, and system and tachometer feedback gains are varied. The curves show that tachometer feedback alone is capable of stabilizing this system, for $K < 1.87 \times 10^7$. For system gains greater than this value, acceleration feedback is required to return the zeros to the

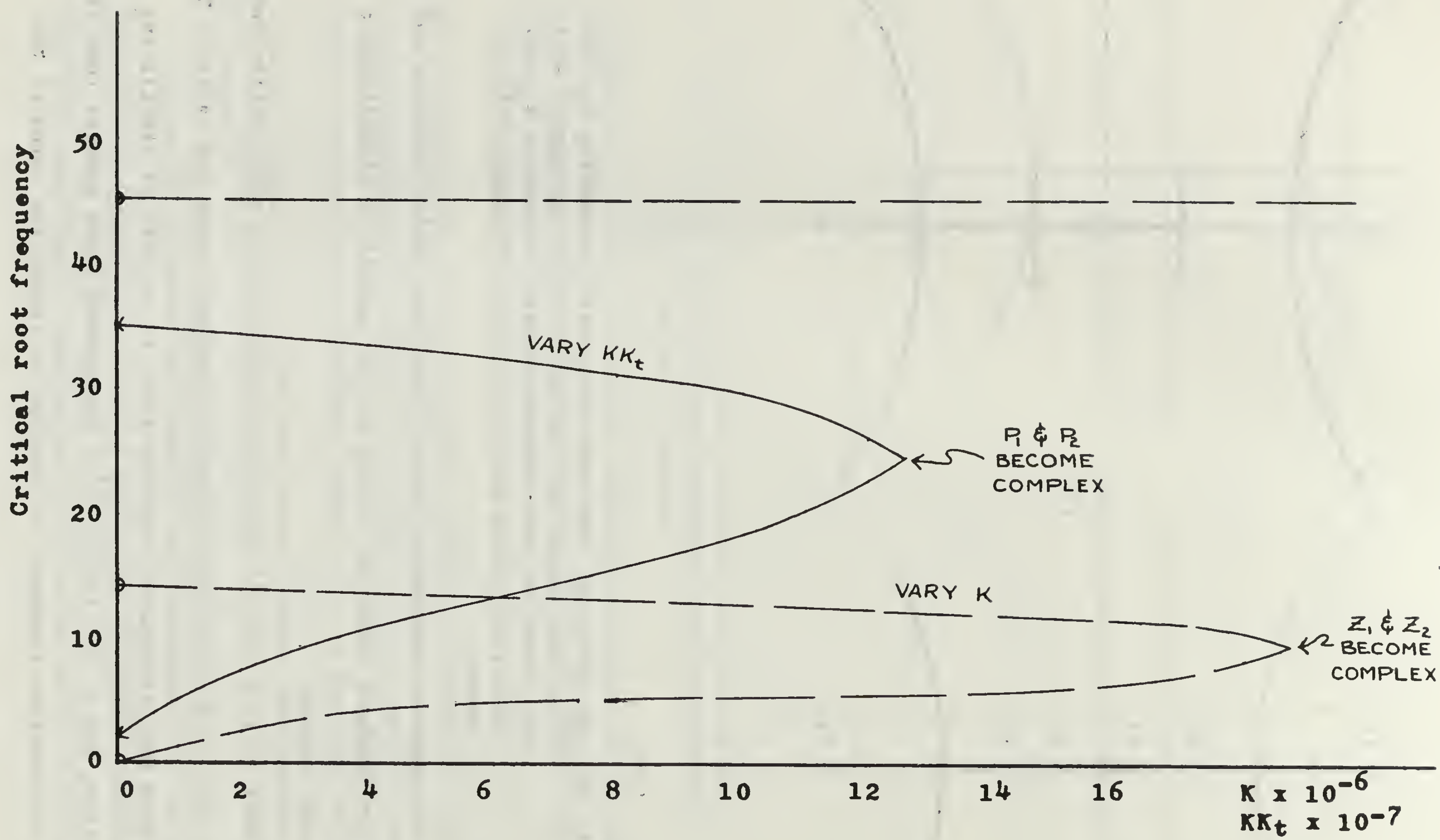


Fig. 31. Pole-zero movements of 6^{th} order system for varying system and tachometer feedback gains.

imaginary axis.

Figure 32 is a plot of pole-zero movements for varying tachometer and acceleration feedback gains. The curves in this figure were generated from root locus calculations. The curves show that acceleration feedback alone is not capable of stabilizing this system, except at lower values of system gain. For high values of system gain, Z_2 and Z_3 become complex before Z_1 is moved far enough to stabilize the system. The curves indicate, however, that a combination of tachometer and acceleration feedback should normally be capable of stabilizing a sixth order system, and that tachometer feedback alone is capable of stabilizing this system.

Figure 33 includes curves from the family of tachometer feedback stability curves. The area below the curves is the stable region, when the indicated values of tachometer feedback gain are used. The curves terminate with $KK_a = 8.6 \times 10^5$, due to Z_2 and Z_3 becoming complex for larger amounts of acceleration feedback.

Root loci are plotted in Figures 34 and 35. Figure 34 is the root locus for the example system stabilized with tachometer feedback only. While the system is obviously stable, the roots are quite close to the imaginary axis, and the response will be very oscillatory. The root location on the locus is due to the small root locus gain available to the normal sixth order system. This gain is the reciprocal of the coefficient of the S^5 term in the characteristic equation. This coefficient is not affected by tachometer or acceleration feedback; therefore, good response can not be achieved with this type of compensation.

Figure 35 is the root locus of the example system, using both tachometer and acceleration feedback compensation. Again, the system is stabilized; however, the response has not been improved.

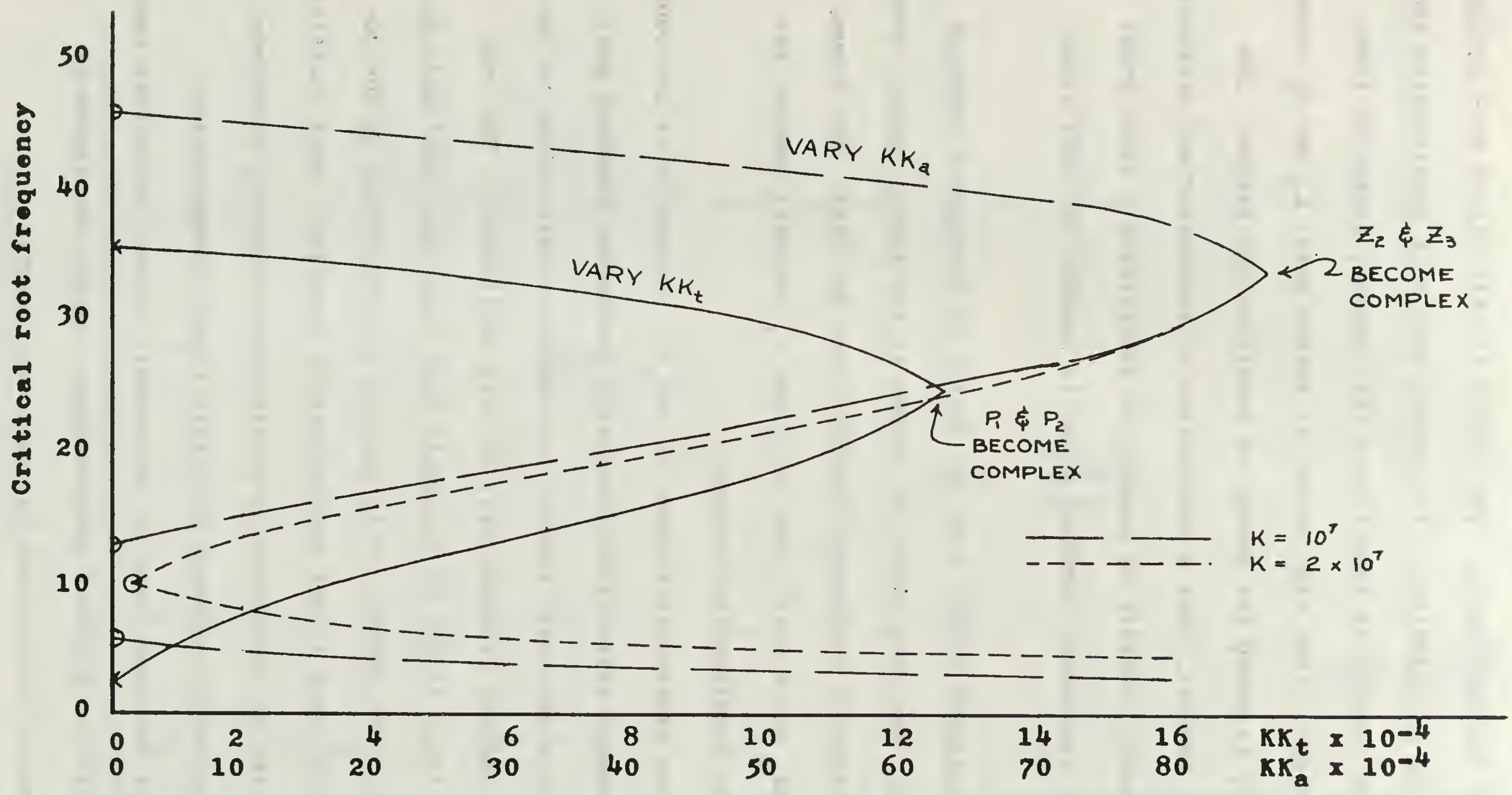


Fig. 32. Pole-zero movements of 6^{th} order system for varying tachometer and acceleration feedback gains.

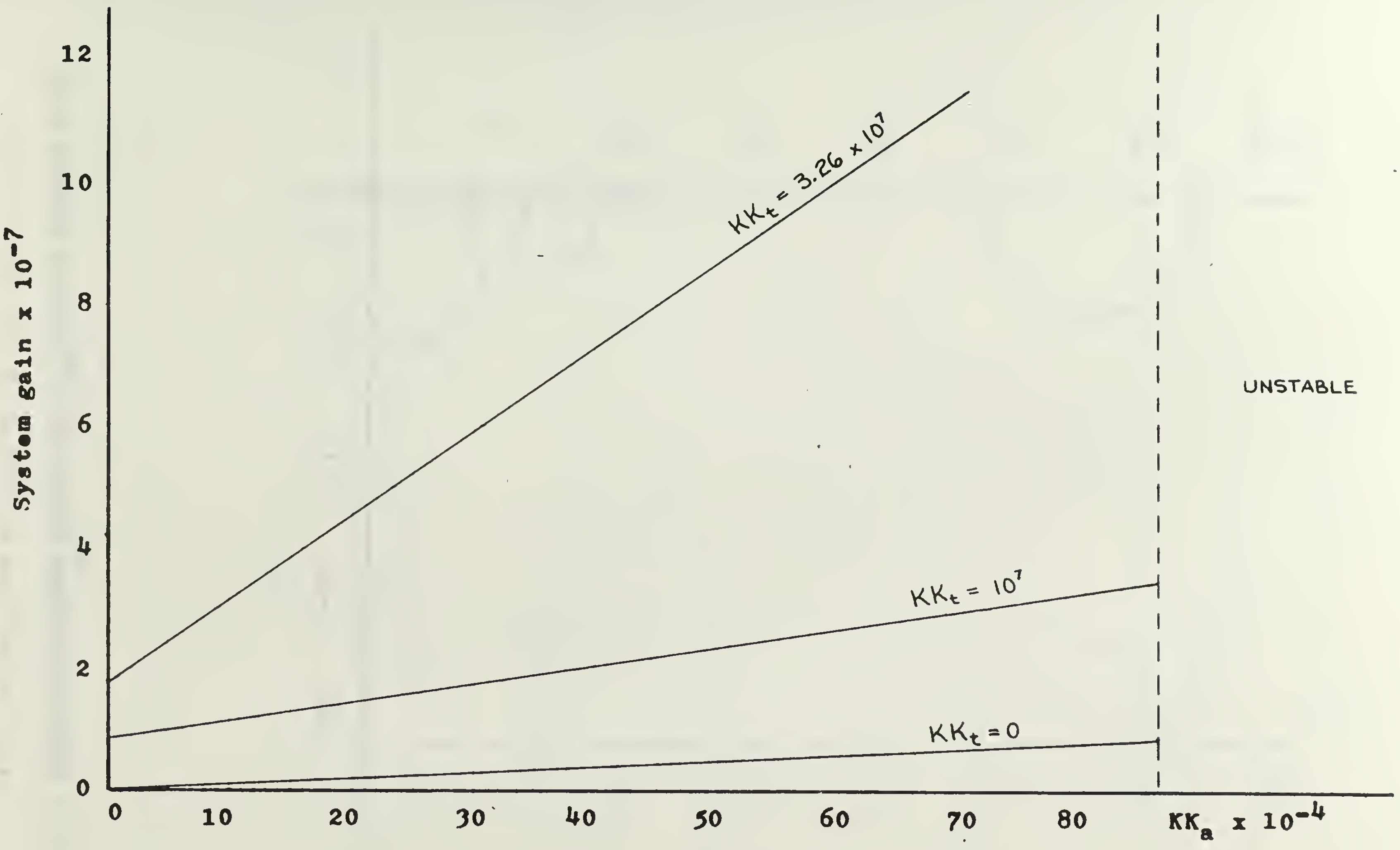


Fig. 33. Stability curves of 6^{th} order system.

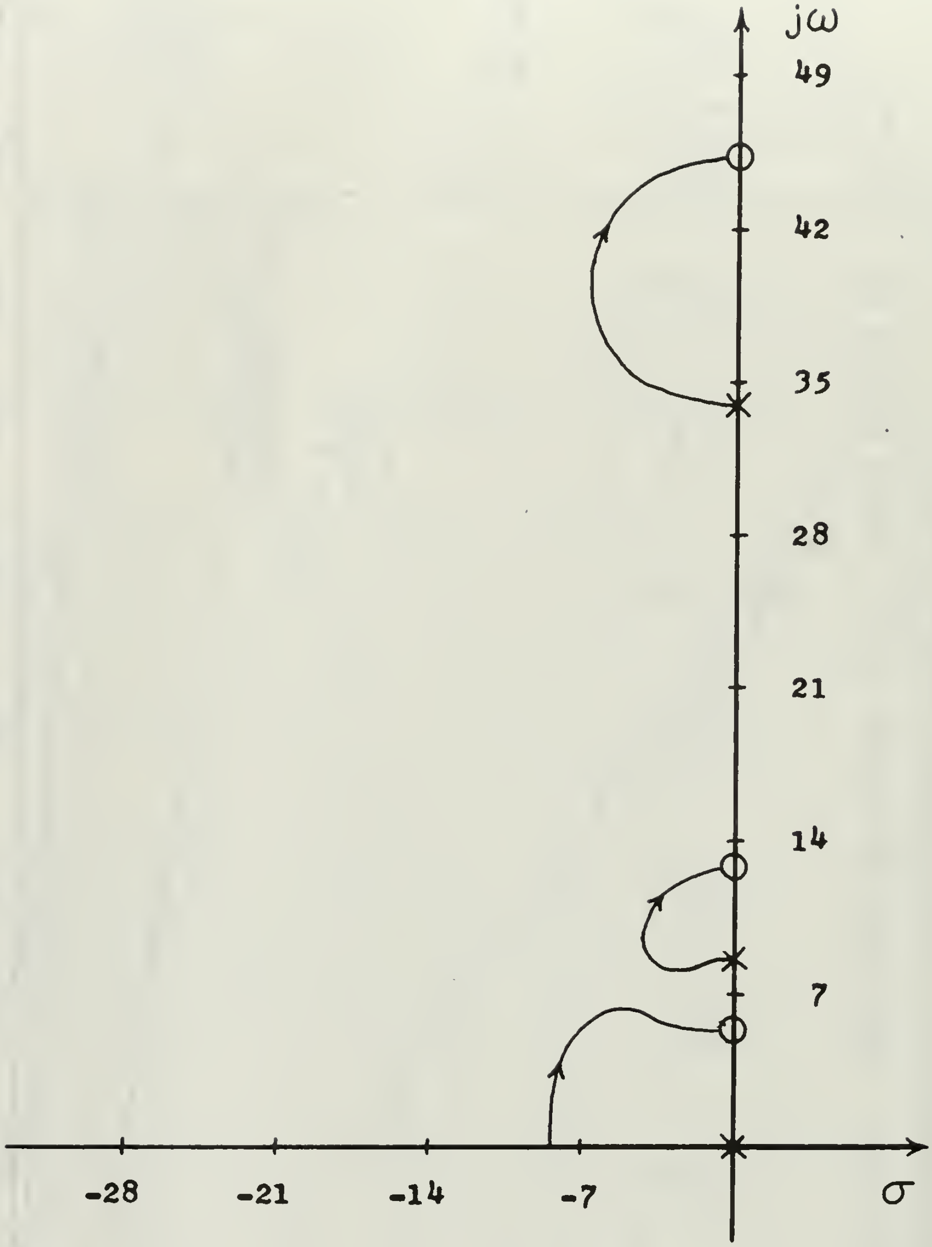


Fig. 34. Root locus of 6^{th} order system with $K_t = 0.008$ and $K_a = 0.0$.

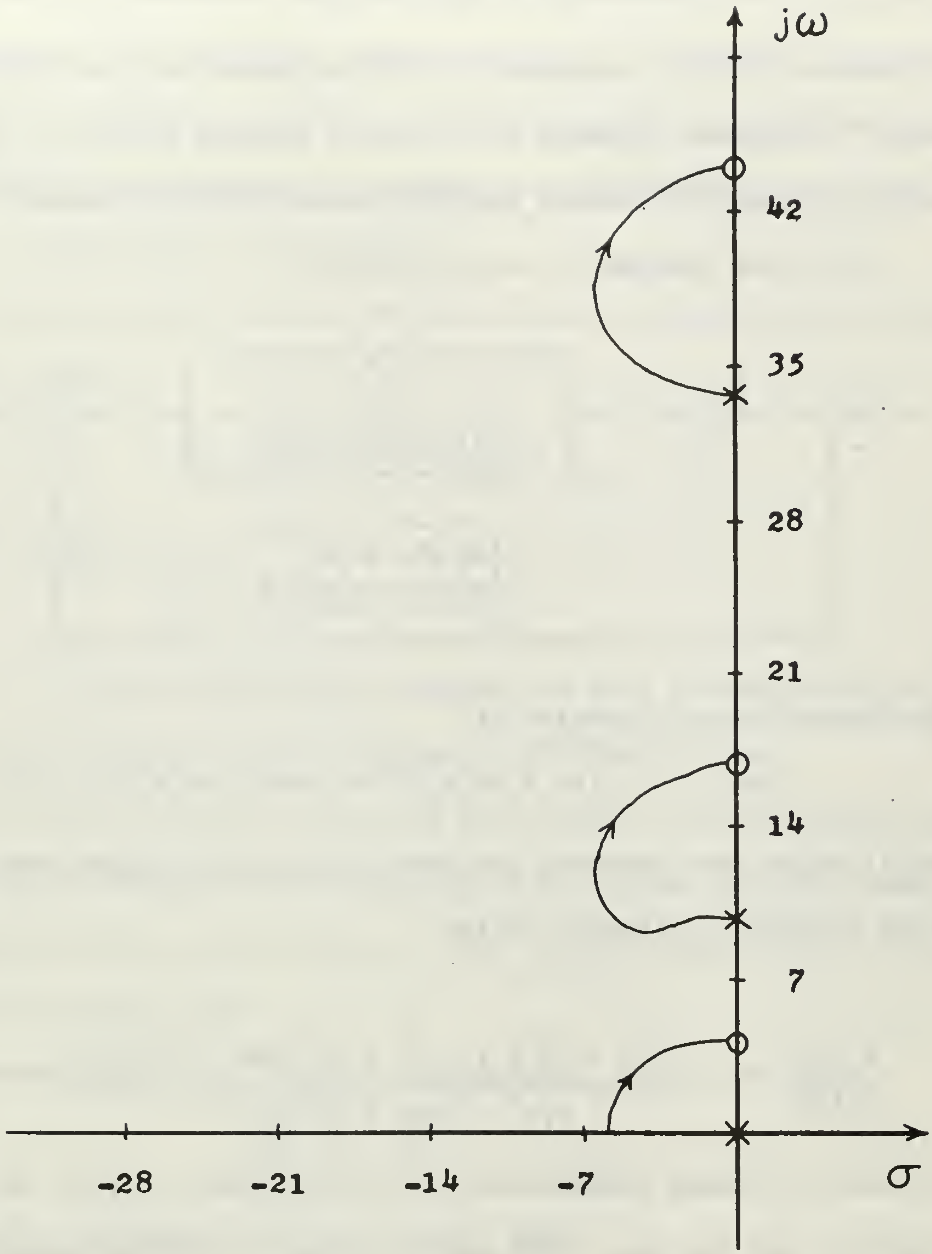


Fig. 35. Root locus of 6^{th} order system with $K_t = 0.01$ and $K_a = 0.02$.

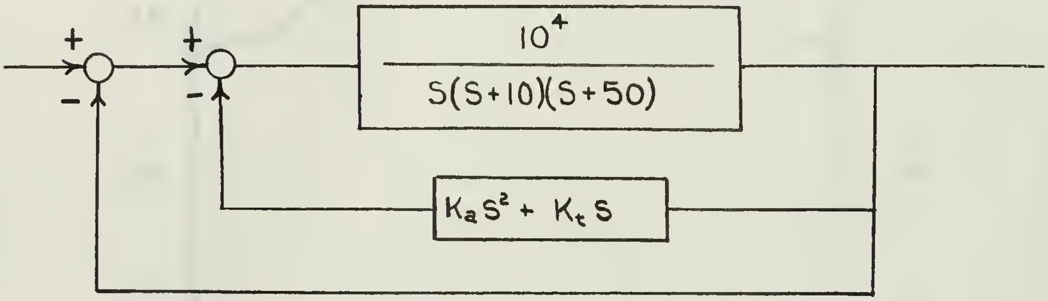
6. Design of compensation for a third order system.

The system to be compensated has the transfer function

$$G(S) = \frac{10^4}{S(S + 10)(S + 50)} .$$

The system is to be compensated with tachometer and/or acceleration feedback. The system response is to have a damping factor of $\zeta = 0.50$, and is to have the maximum possible velocity-error constant.

The block diagram of this system is



The characteristic equation is

$$F(S) = S^3 + (60 + 10^4 K_a)S^2 + (500 + 10^4 K_t)S + 10^4 .$$

Partitioning this equation and putting it in the proper form for the root locus stability criterion yields

$$\frac{F_e(S)}{F_o(S)} = \frac{(60 + 10^4 K_a) \quad [S^2 + 10^4/(60 + 10^4 K_a)]}{S[S^2 + (500 + 10^4 K_t)]} = -1 .$$

In order to design compensation, it is necessary to vary the pole-zero locations and the root locus gain to place the system roots in proper position, in the S-plane, to give the required damping. For the third order case, the pole-zero locations are obvious from inspection of the root locus equation, when values are substituted for K_a and K_t. The root locus gain is the coefficient of the S^2 term in the characteristic

equation.

The design of compensation must satisfy two requirements which are not independent. It is expected that the requirement that $\zeta = 0.5$ could be met with numerous combinations of tachometer and acceleration feedback gains. The velocity error constant, K_v, depends on the value of tachometer feedback gain used. Therefore, it is expected that there is some optimum combination of tachometer and acceleration feedback, which will satisfy both requirements.

It has been shown that the velocity-error constant can be determined as follows. [3]

$$1/K_v = \frac{d}{ds} \left[\frac{1}{1 + F(S)} \right]_{S=0}$$

For this system,

$$K_v = 10^8/(500 + 10^4 K_t).$$

Therefore, for the velocity-error constant to be a maximum requires that the minimum possible amount of tachometer feedback be used.

Use of this root locus stability criterion in design requires some knowledge of the shape of the root locus for a given pole-zero combination. Figures 36(a) and (b) show the two extreme forms which the third order root loci can take.

In general, the root loci in Figure 36(a) will occur when

$$|Z_1| > |P_1 - Z_1|.$$

The root loci in Figure 36(b) will apply when

$$|Z_1| \ll |P_1 - Z_1|.$$

For pole-zero locations other than above, the root loci will take the general shape shown in Figure 36(c).

If there is an optimum combination of tachometer and acceleration

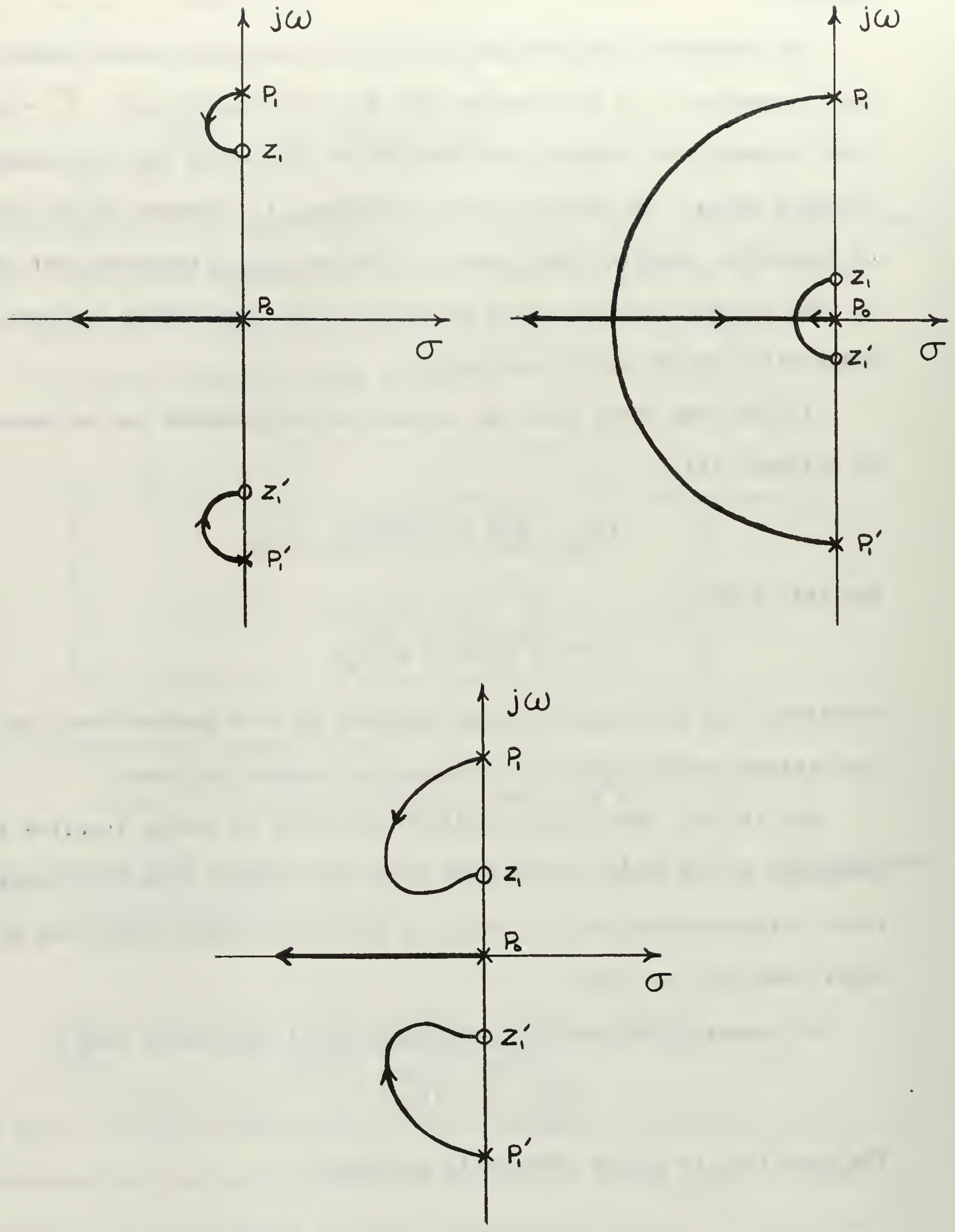


Fig. 36. (a). Root locus for $|Z_1| > |P_1 - Z_1|$. (b). Root locus for $|Z_1| \ll |P_1 - Z_1|$. (c) Root locus for $|Z_1| < |P_1 - Z_1|$.

feedback, to meet the design requirements, a plot of velocity-error constant versus acceleration feedback gain, for $\zeta = 0.5$, should show a maximum. To plot such a curve accurately would require a great deal of effort; however, using the information in Figures 36(a), (b) and (c) should allow the plotting of a reasonable approximation to the true curve. Points on the approximate curve will be generated with the following procedure:

1. Assume a value of acceleration feedback gain to locate Z_1 in the S-plane.
2. Sketch in the approximate root locus crossing the $\zeta = 0.5$ line, and entering Z_1.
3. By trial and error, locate P_1 for root locus gain = $(60 + 10^4 K_a)$.
4. After locating P_1 verify general shape of root locus.
5. Determine the required tachometer feedback gain, and the resulting velocity-error constant.

This procedure is demonstrated in Figure 37. A study of this figure indicates that there is one trivial solution; compensation with acceleration feedback gain very large, causing Z_1 to approach the origin. Since this is not a practical solution to the design, it will not be considered further.

Figure 38 is the plot of velocity-error constant versus acceleration feedback gain resulting from the above procedure. It is seen that the maximum velocity-error constant is achieved by using tachometer feedback alone, with $K_t = 0.0394$. To prove system performance, the root locus of the compensated system is shown in Figure 39. The dominant root is shown, and it is seen that the specified damping factor has

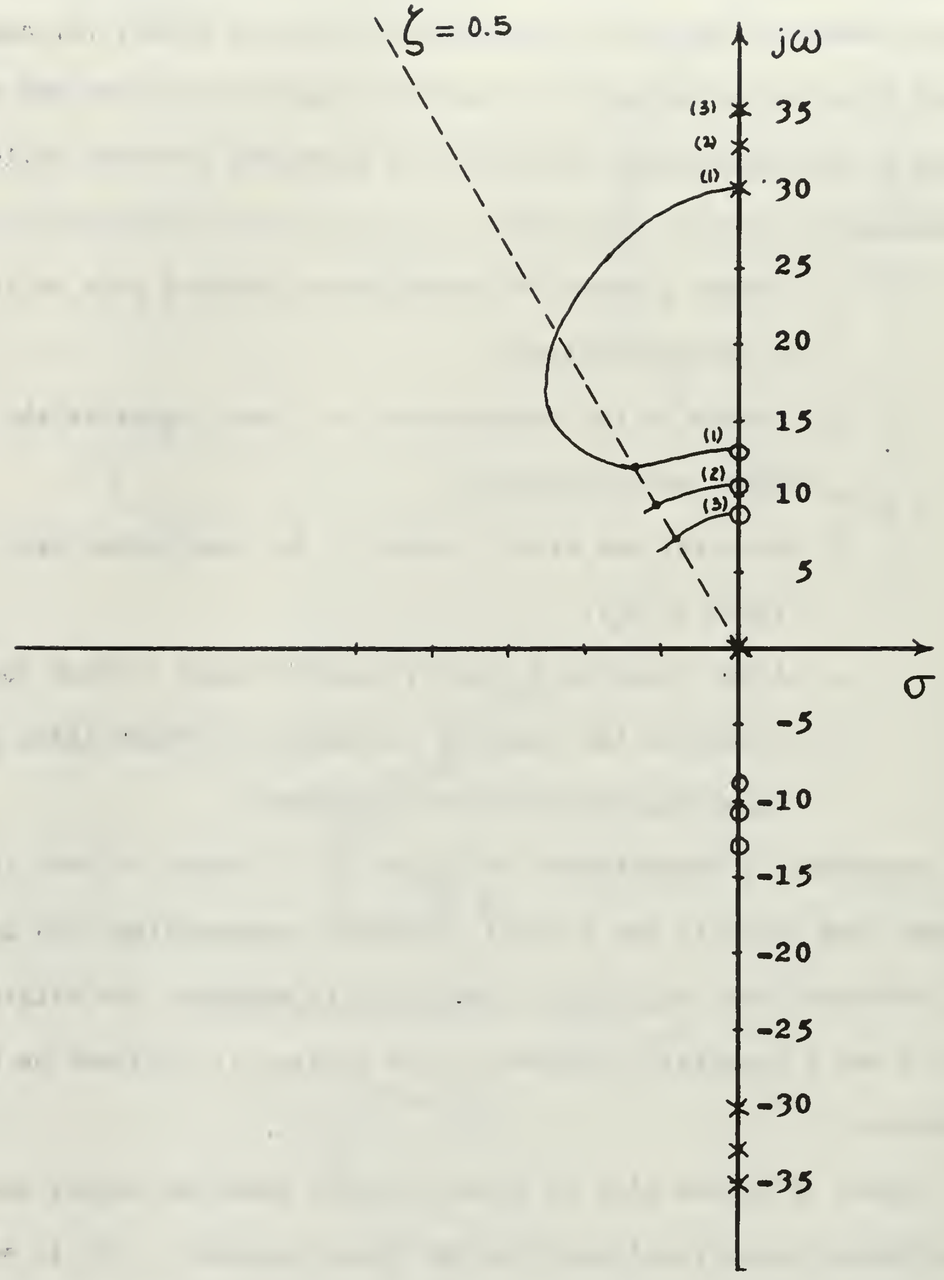


Fig. 37. Root locus of 3rd order system used to generate points on the approximate velocity-error constant curve.

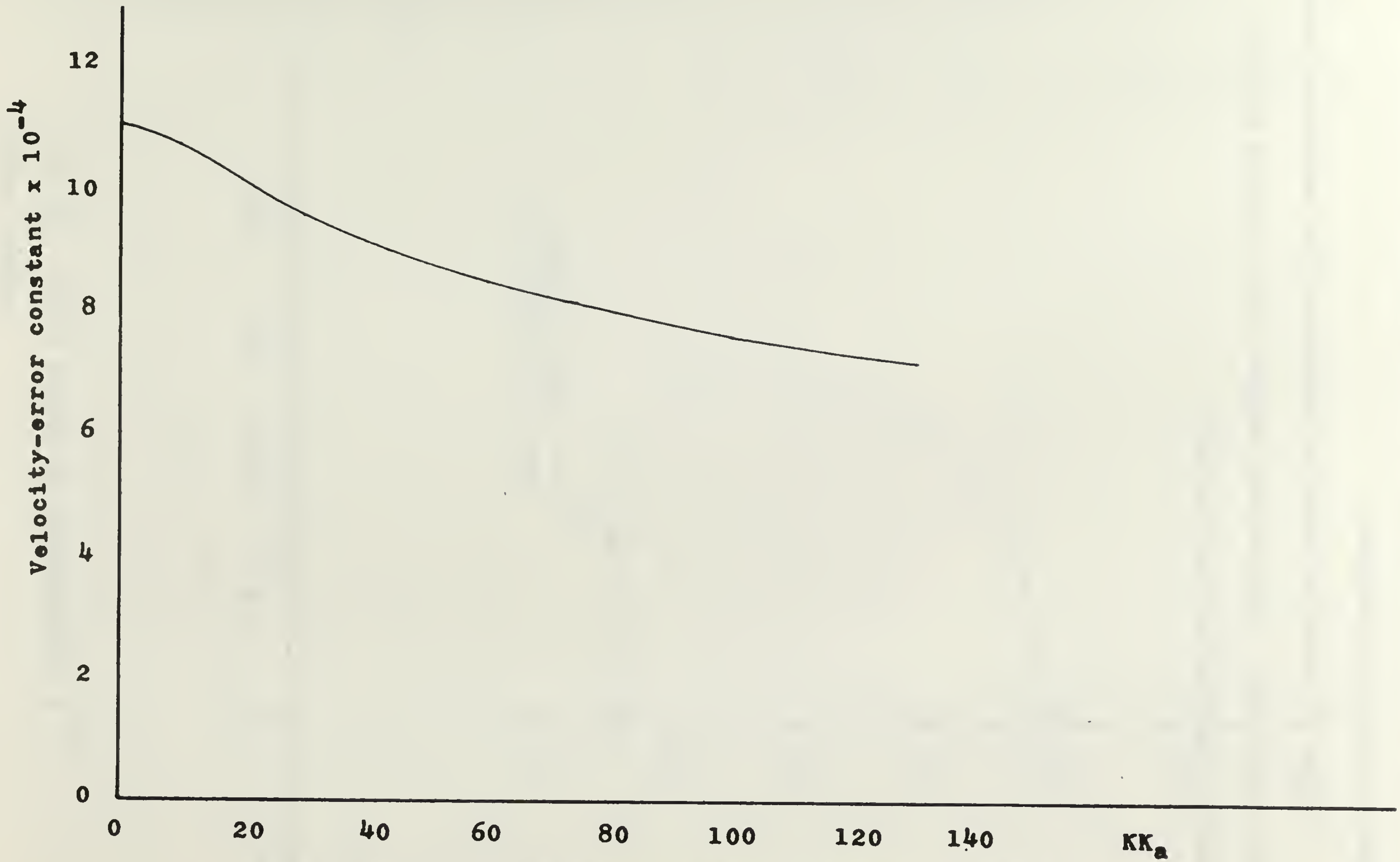


Fig. 38. Plot of velocity-error constant vs acceleration feedback gain. 3^{rd} order system with = 0.5.

been realized. Since the minimum tachometer feedback has been used to achieve the required damping, the maximum possible velocity-error constant has also been realized.

Figure 40 shows the transient response of the compensated system, for a step input. The transient response curve verifies that the specified damping has been realized.

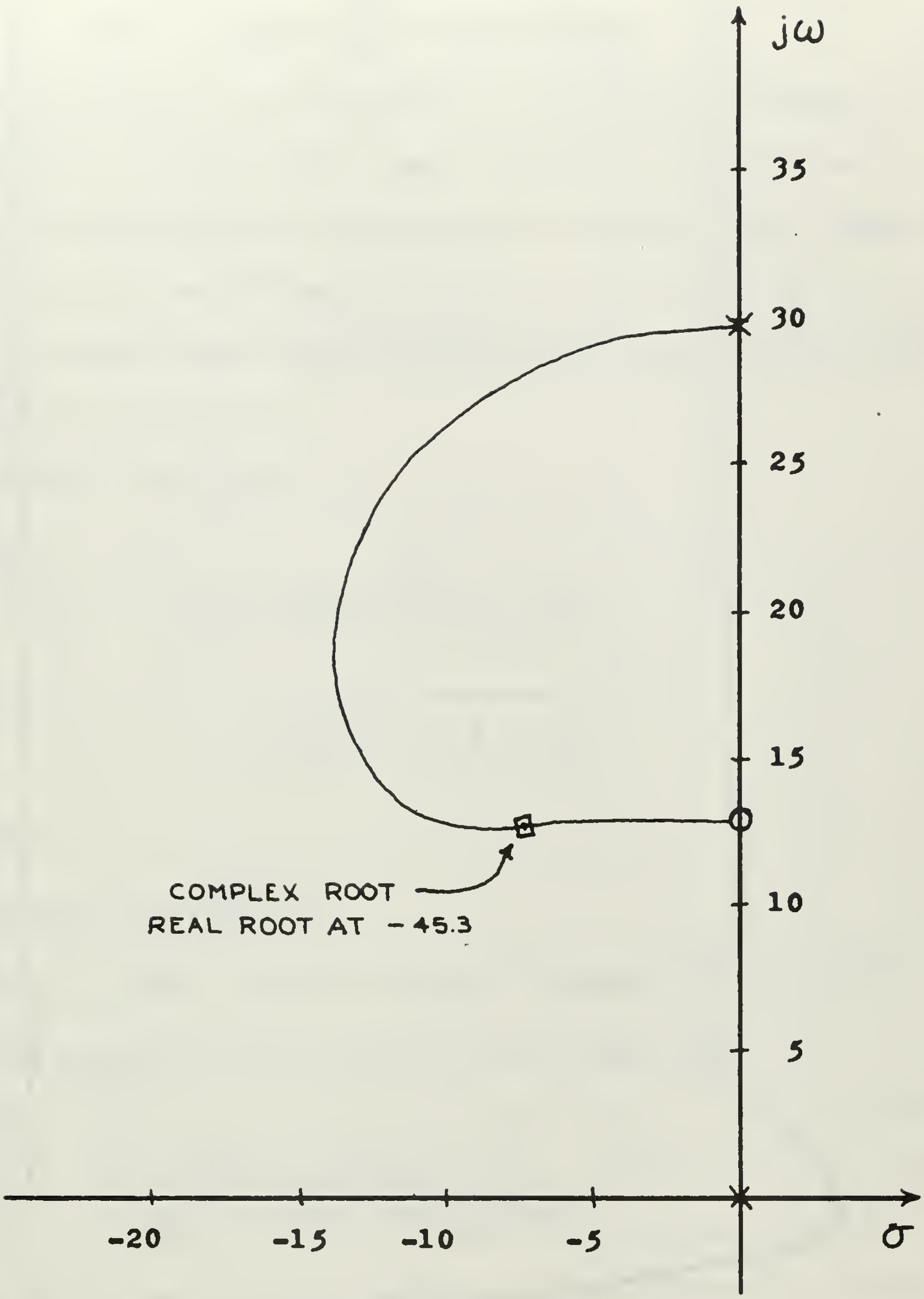


Fig. 39. Root locus of compensated 3rd order system.

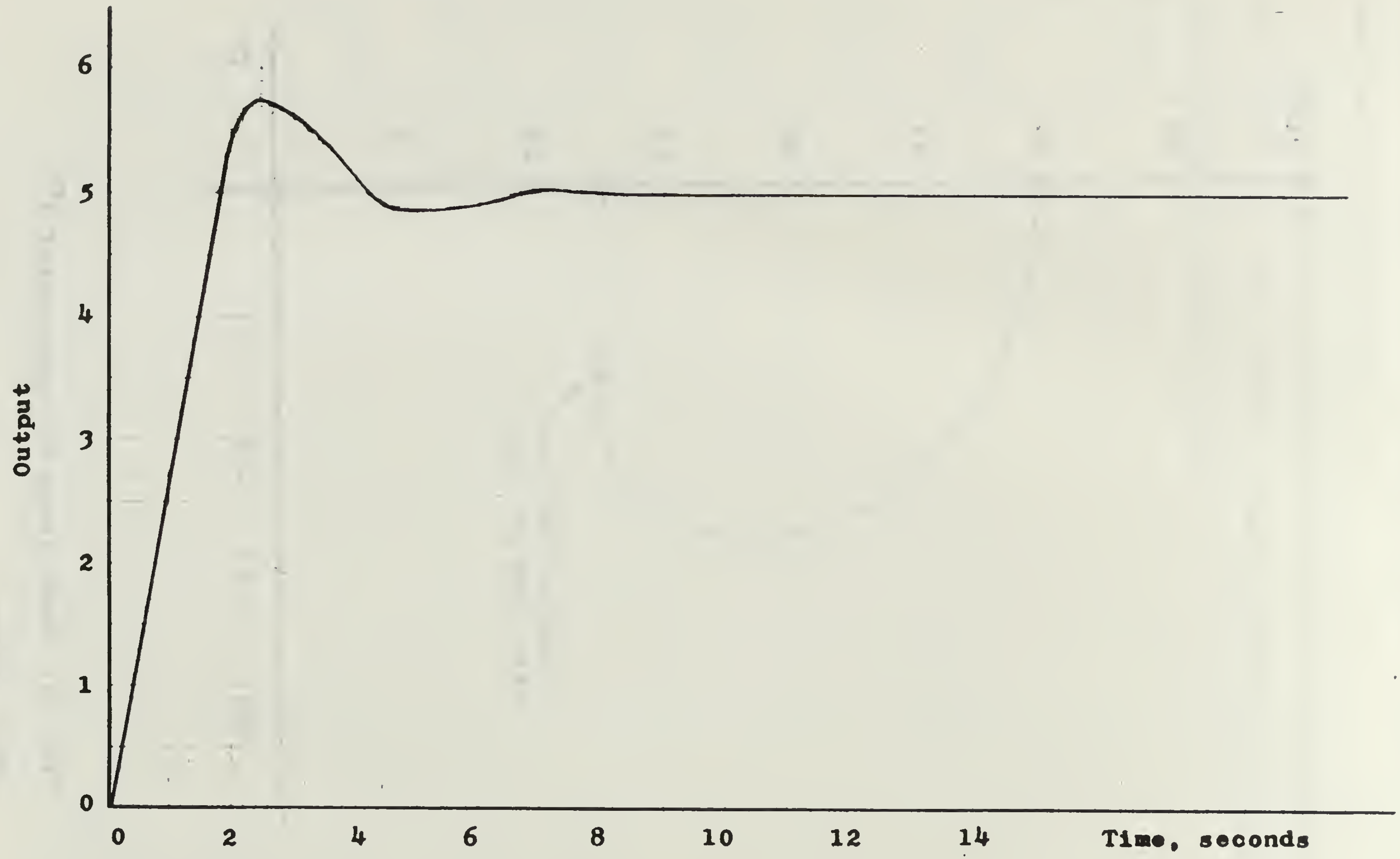


Fig. 40. Transient response of compensated 3^{rd} order system.

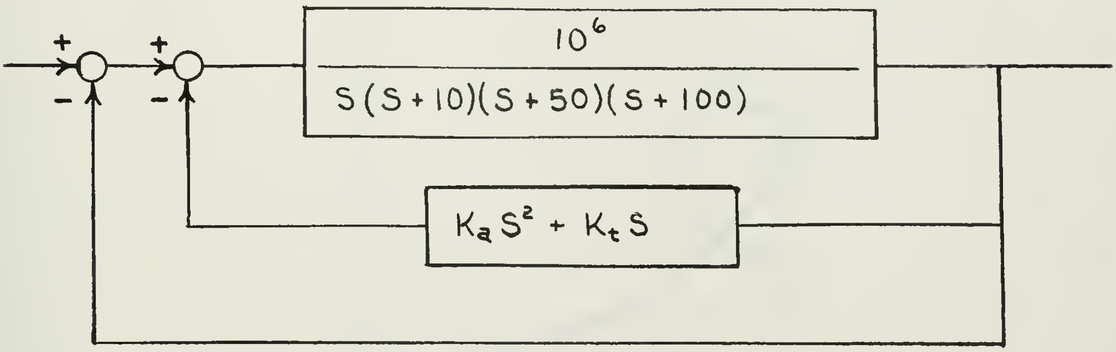
7. Design of tachometer and acceleration feedback compensation for a fourth order system.

The system to be compensated has the following transfer function

$$G(S) = \frac{10^6}{S(S + 10)(S + 50)(S + 100)}$$

It is to be compensated with tachometer and acceleration feedback to provide transient response with a damping factor of $\zeta = 0.5$, and is to have the maximum possible velocity error constant. This sytem is the third order system of Chapter 6, with a pole added and the gain increased. The design specifications are the same, and a similar design technique will be attempted.

The system block diagram is



The characteristic equation is

$$F(S) = S^4 + 160S^3 + (5150 + 10^6K_a)S^2 + (50000 + 10^6K_t)S + 10^6 = 0.$$

Putting this equation in the proper root locus form yields

$$\frac{F_e(S)}{F_o(S)} = \frac{S^4 + (5150 + 10^6K_a)S^2 + 10^6}{160S[S^2 + (50000 + 10^6K_t)/160]} = -1.$$

In general, this root locus will appear as shown in Figure 41. It is seen that a second order approximation may be used to design for the required response of the system. If however, P_1 is moved up the imaginary axis far enough, the root locus will appear as shown in Figure

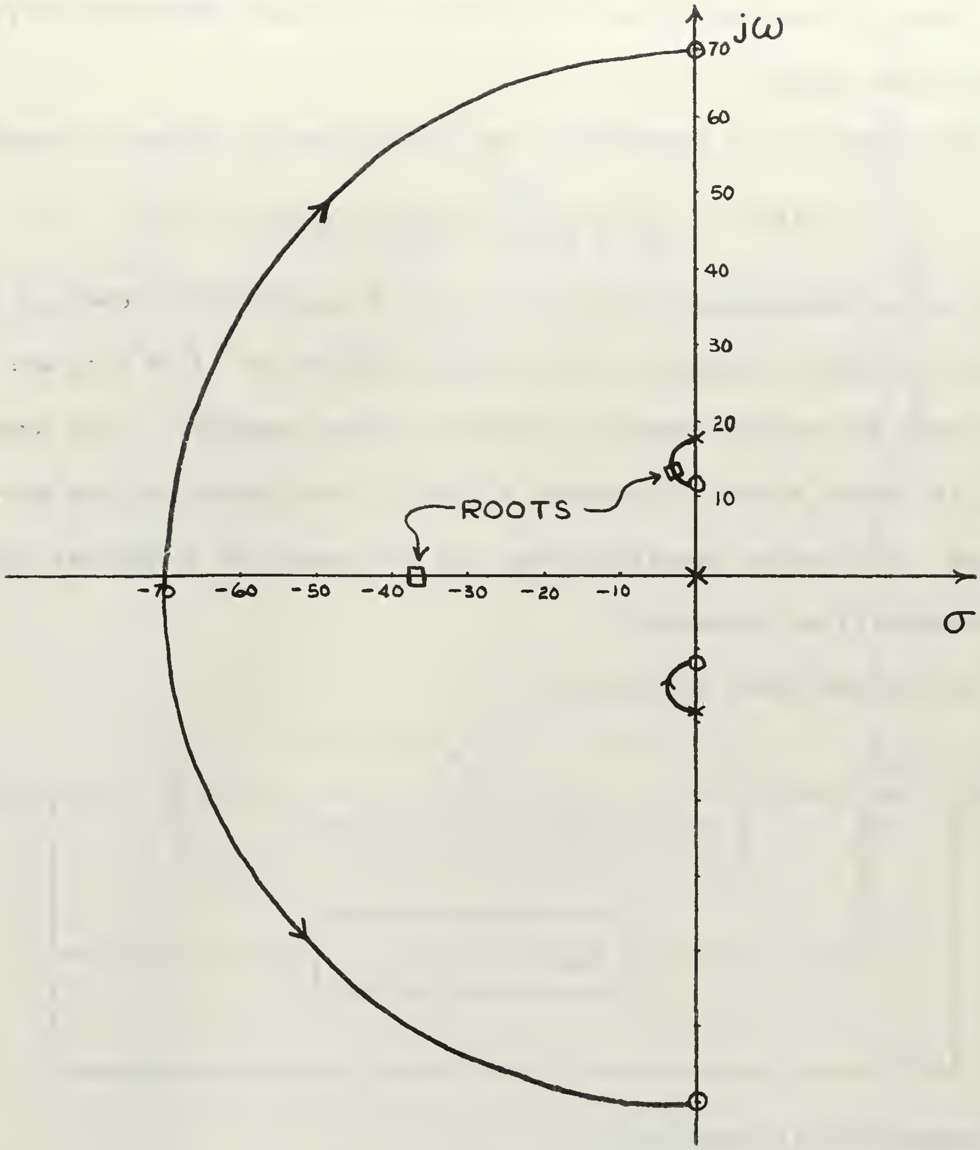


Fig. 41. Root locus of uncompensated 4^{th} order system.

42. The real root would lie close to the origin in this case, and the second order dominance approximation could not be applied. Therefore, it is desired to move P_1 and Z_1 within the limits necessary to assure the general root locus form in Figure 41.

It can be shown that the velocity-error constant for this system is

$$K_v = 1/(K_t + .05)$$

In order to achieve the maximum velocity-error constant, it is desirable

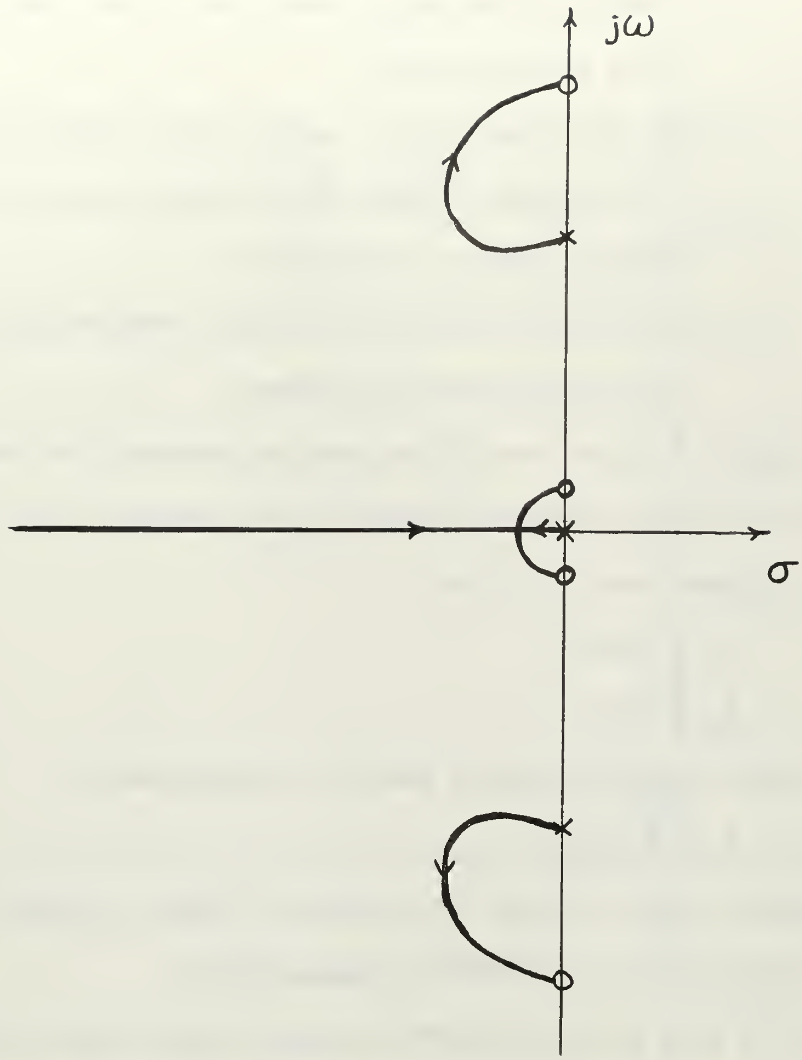


Fig. 42. Root locus of fourth order system with excessive tachometer and acceleration feedback. Small negative real root dominant.

to keep the tachometer feedback gain as low as possible.

The compensation design will be accomplished in a manner similar to that used for the third order system. An approximate curve of velocity-error constant versus acceleration feedback gain, for $\zeta = 0.5$, will be plotted to determine the required compensation. Points on the approximate curve will be generated with the following procedure:

1. Assume a value of acceleration feedback gain to locate Z_1 in the S-plane.

2. Sketch in the approximate root locus crossing the $\zeta = 0.5$ line and entering Z_1.

3. By trial and error, locate P_1 for root locus gain = 1/160.

4. Verify general shape of the root locus, and check for second order root dominance.

5. Determine required tachometer feedback gain and the resulting velocity-error constant.

Figure 43 is the root locus used to generate the above points. Figure 44 is the resulting velocity-error constant curve. It is seen that a maximum does occur for

$$K_a = 0.0038$$

$$K_v = 11.0.$$

This value of velocity-error constant corresponds to

$$K_t = 0.0409.$$

Figure 45 is the root locus for the system with the indicated tachometer and acceleration feedback compensation.

It is seen that the system has approximate second order root dominance. The compensated system has a damping factor of $\zeta = 0.47$, quite close to the specified value. This indicates that the design technique used is valid. Figure 46 is a plot of the transient response of the compensated system. This curve verifies that the designed compensation is adequate.

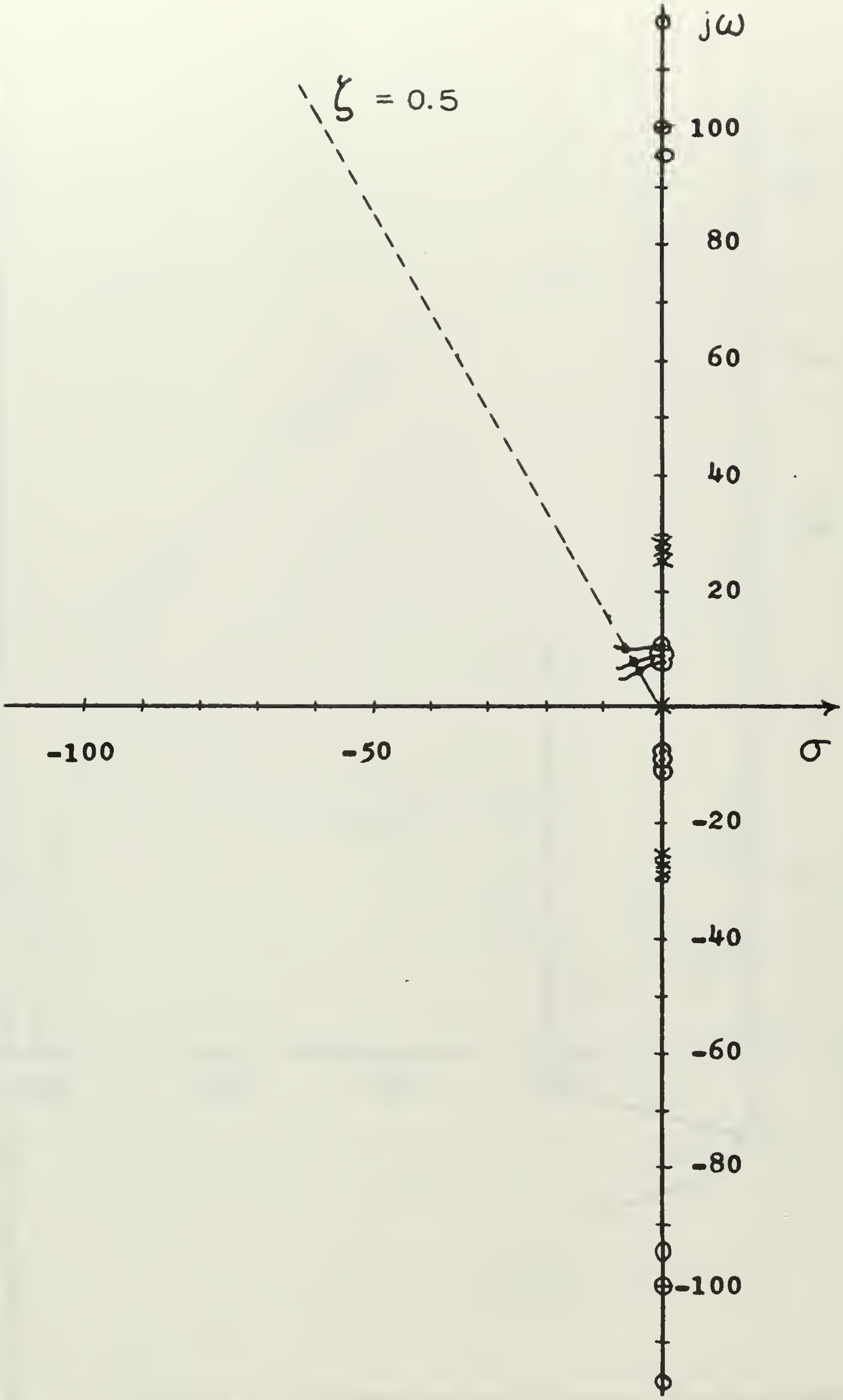


Fig. 43. Root locus of 4th order system used to generate points on approximate velocity-error constant curve.

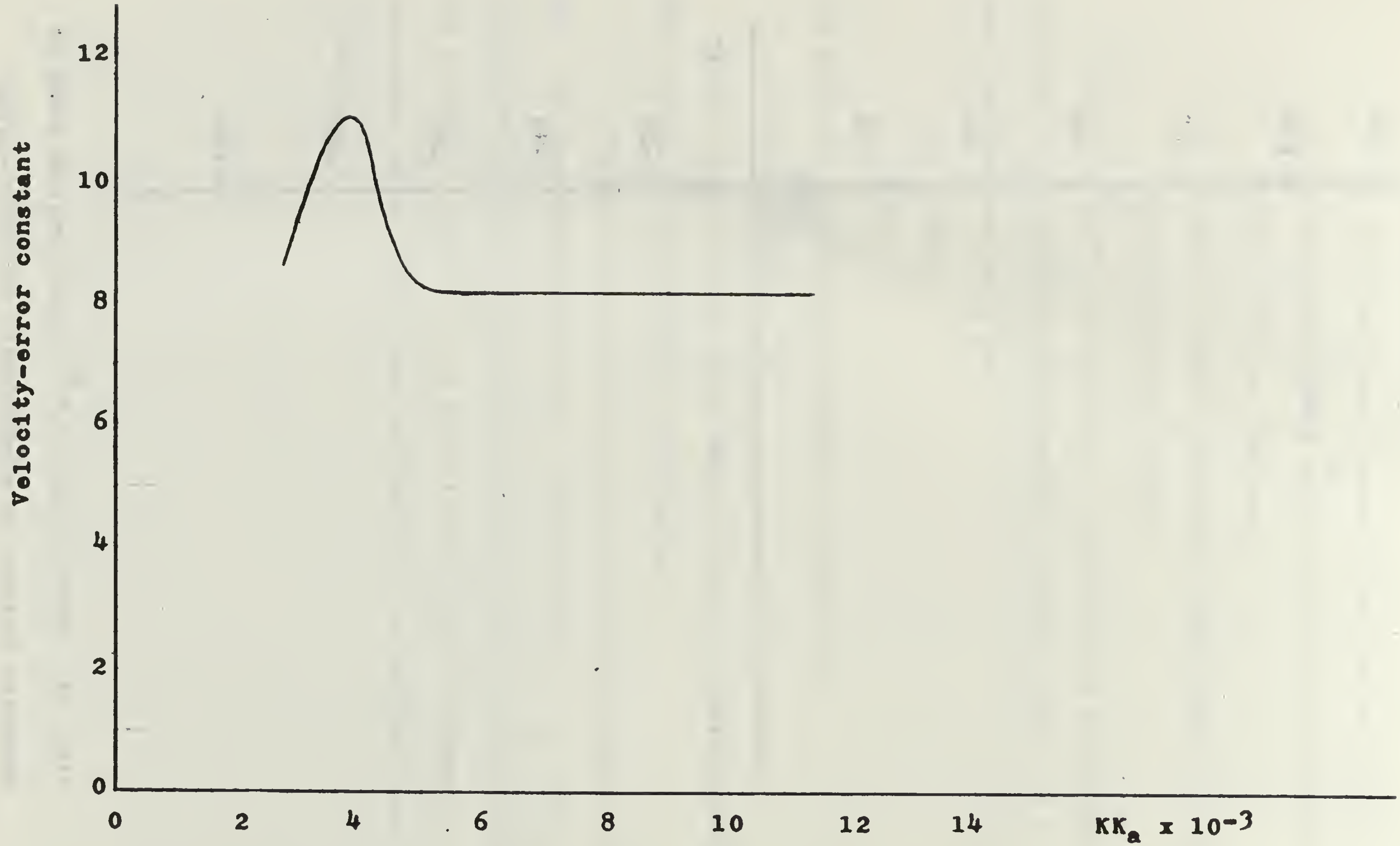


Fig. 44. Plot of velocity-error constant vs acceleration feedback gain. 4^{th} order system with = 0.5.

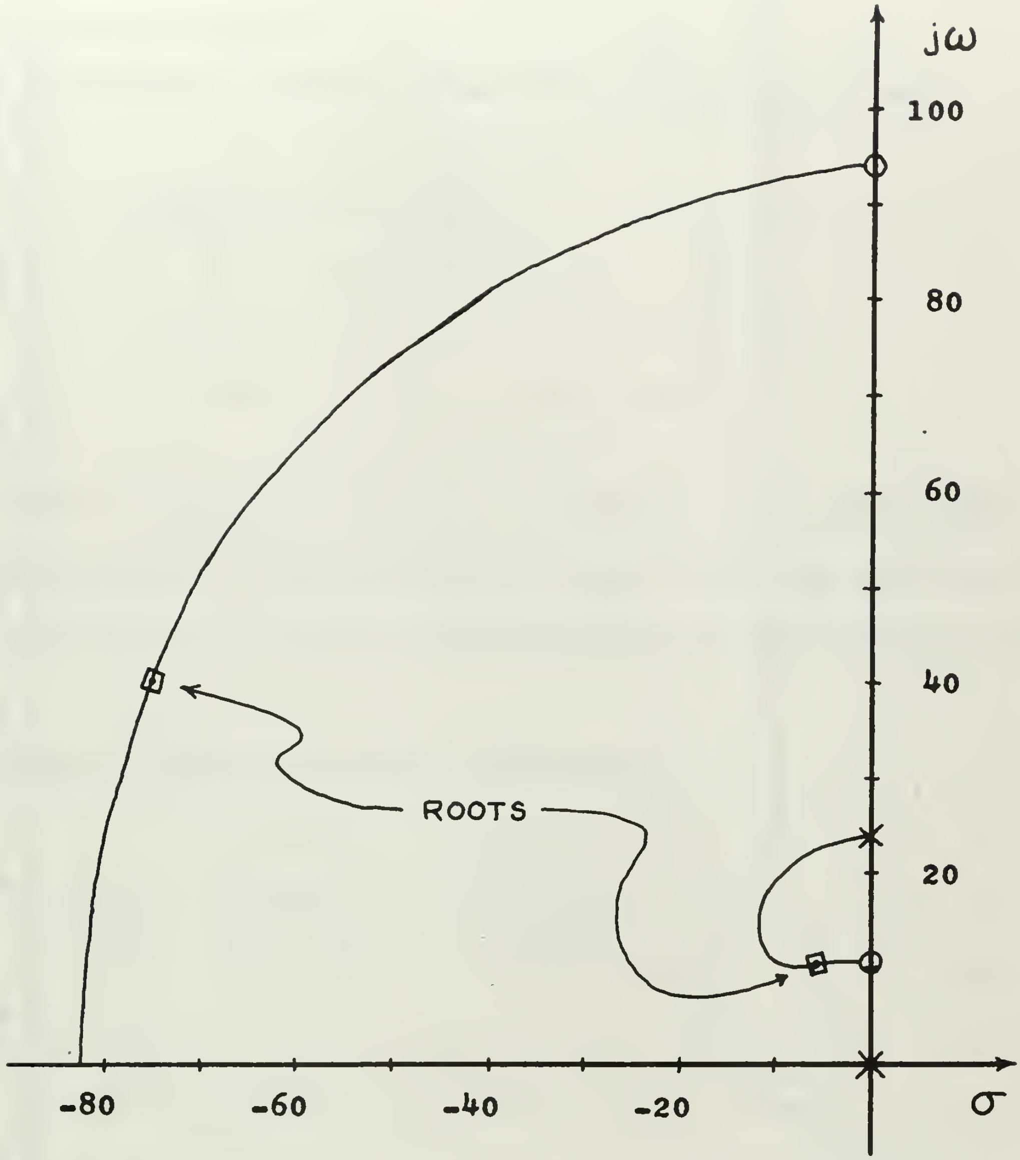


Fig. 45. Root locus of compensated 4^{th} order system

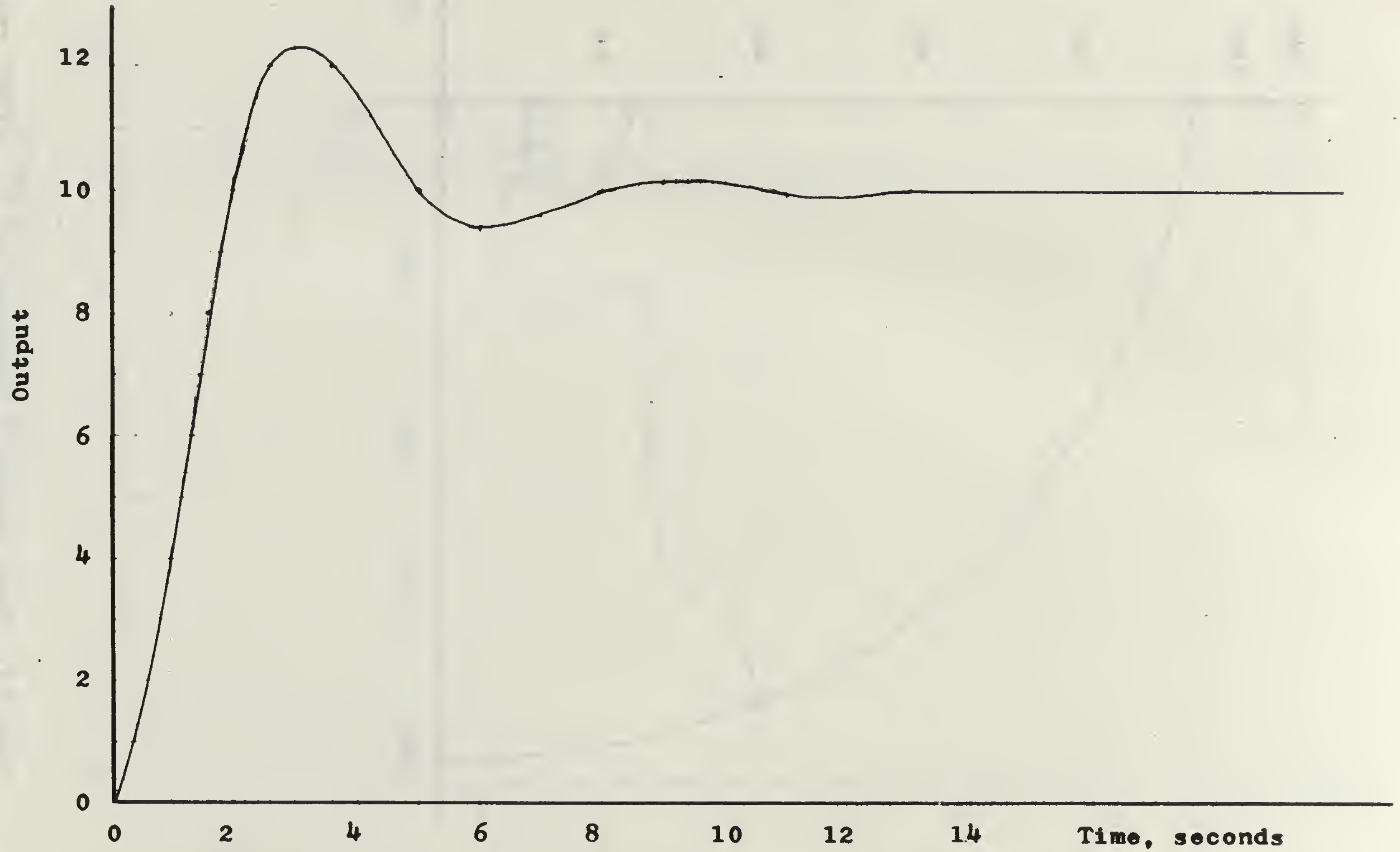
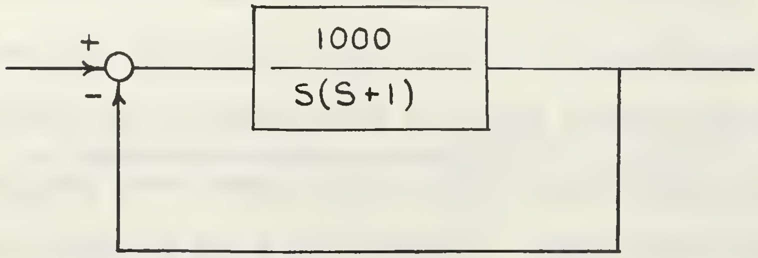


Fig. 46. Transient response of compensated 4^{th} order system.

8. Design of cascade compensation for a second order system.

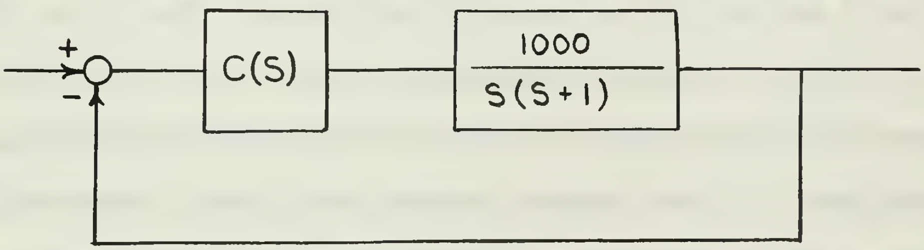
The design of cascade compensation will be attempted in this chapter. The difficulties of designing this type compensation, with the root locus method, will be presented.

The system to be compensated has the following block diagram



This system is stable, but is highly underdamped. It is desired that the peak overshoot be limited to thirty percent for a step input, and that the bandwidth be kept low without reducing the velocity-error constant.

The block diagram for cascade compensation is



With C(S) = (S + z)/(S = p), the system has the following characteristic equation:

$$F(S) = S^3 + (1 + p)S^2 + (1000 + p)S + 1000z = 0$$

It can be shown that

$$K_v = 1000z/p.$$

A phase lag filter is indicated to meet the requirement that the velocity-error constant not be reduced. This type of filter will also meet the

low bandwidth requirement.

The root locus form of the characteristic equation is

$$\frac{F_e(S)}{F_o(S)} = \frac{(1 + p)\ [S^2 + 1000z/(1 + p)]}{S[S^2 + (1000 + p)]} = -1.$$

In order for the filter to be useful, it must take effect at some frequencies less than $\omega = 1$. Therefore, it is seen that the value of p will have very little effect on the location of the root locus poles. It will have only minor effects on the root locus gain and the location of the root locus zeros. The value of z can be used to control the location of the zeros. Therefore, it might be expected that the required filter could be designed with the root locus method.

The compensated system will have the root locus shown in Figure 47. From inspection of this root locus, it is seen that one section of phase lag filter will probably not give the required response. There are two obvious difficulties with attempting the design. The root locus gain is small and can not be changed to any degree. Therefore, the root, on the root locus lobe, will be located close to P_1. This will cause the system to be underdamped. Also due to the low root locus gain, the system will not have dominant second order roots. There will be a negative real root close to the origin, which must be considered in the transient response.

Figure 48 is the root locus of the system compensated with the filter

$$C(S) = \frac{(S + 0.5)}{(S + 0.05)}.$$

The roots are shown, and illustrate the problems discussed above.

Additional sections of filter are indicated, and Figure 49 shows the root locus of the system with two identical sections of the above

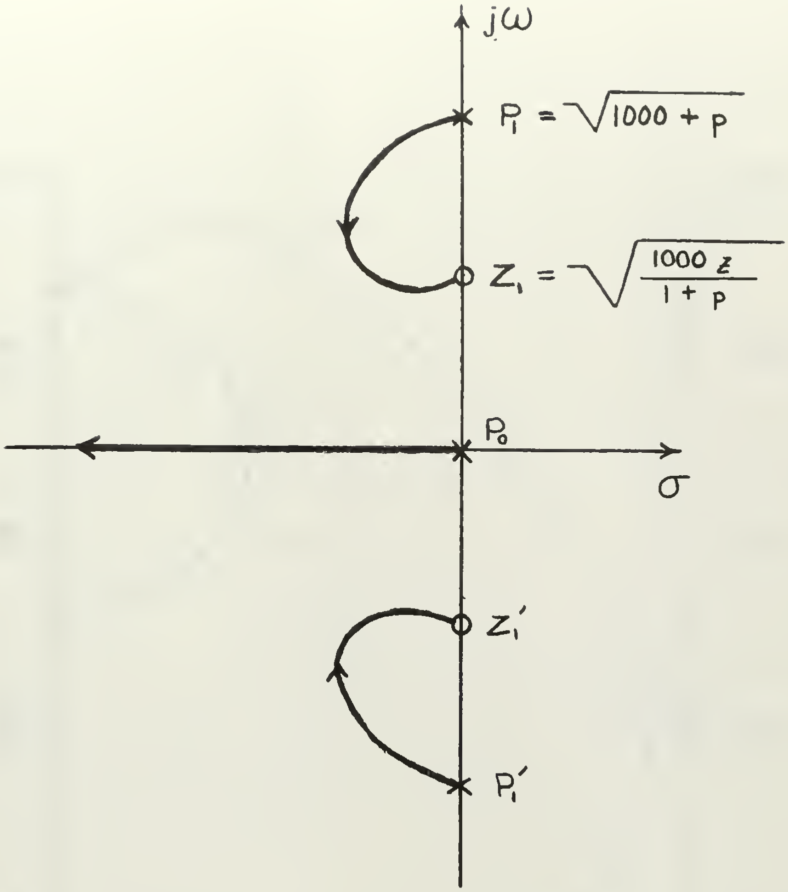


Fig. 47. Generalized root locus of cascade compensated 2^{nd} order system.

filter placed in cascade. It is observed that the order of the basic system has now been doubled. The roots are still very near the imaginary axis, and there are now two pairs of complex roots to consider, as neither pair is dominant. The root loci become increasingly complex as additional sections of filter are added. Since there is not a dominant pair of roots, all roots must be considered in designing for the transient response. It is impossible, practically, to design the compensation by root locus methods.

Although, in specific cases it may be possible to design cascade filters, it is concluded that, in general, this root locus method is not an effective tool for the design of cascade compensation.

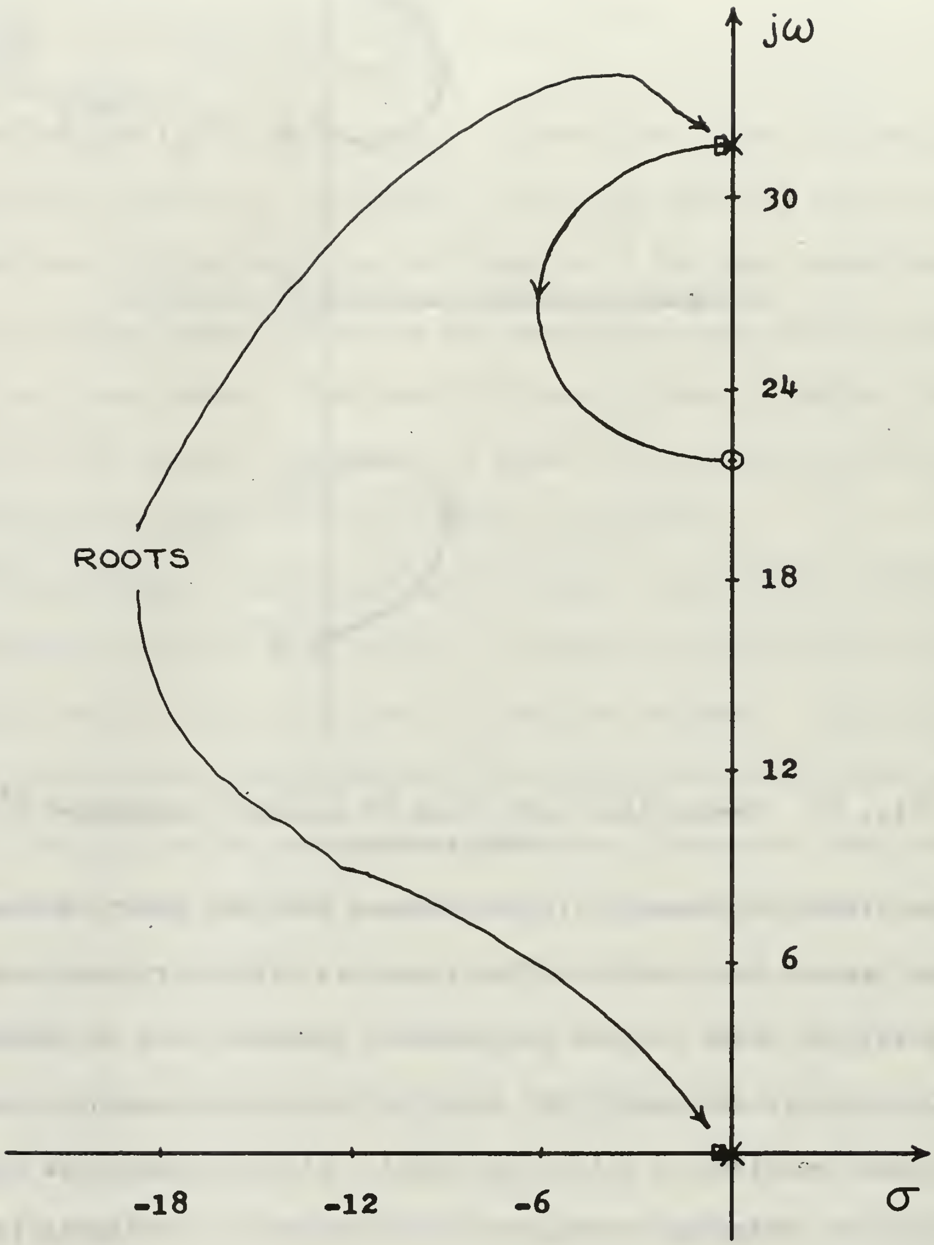


Fig. 48. Root locus of 2nd order system with one section of cascade, phase-lag filter.

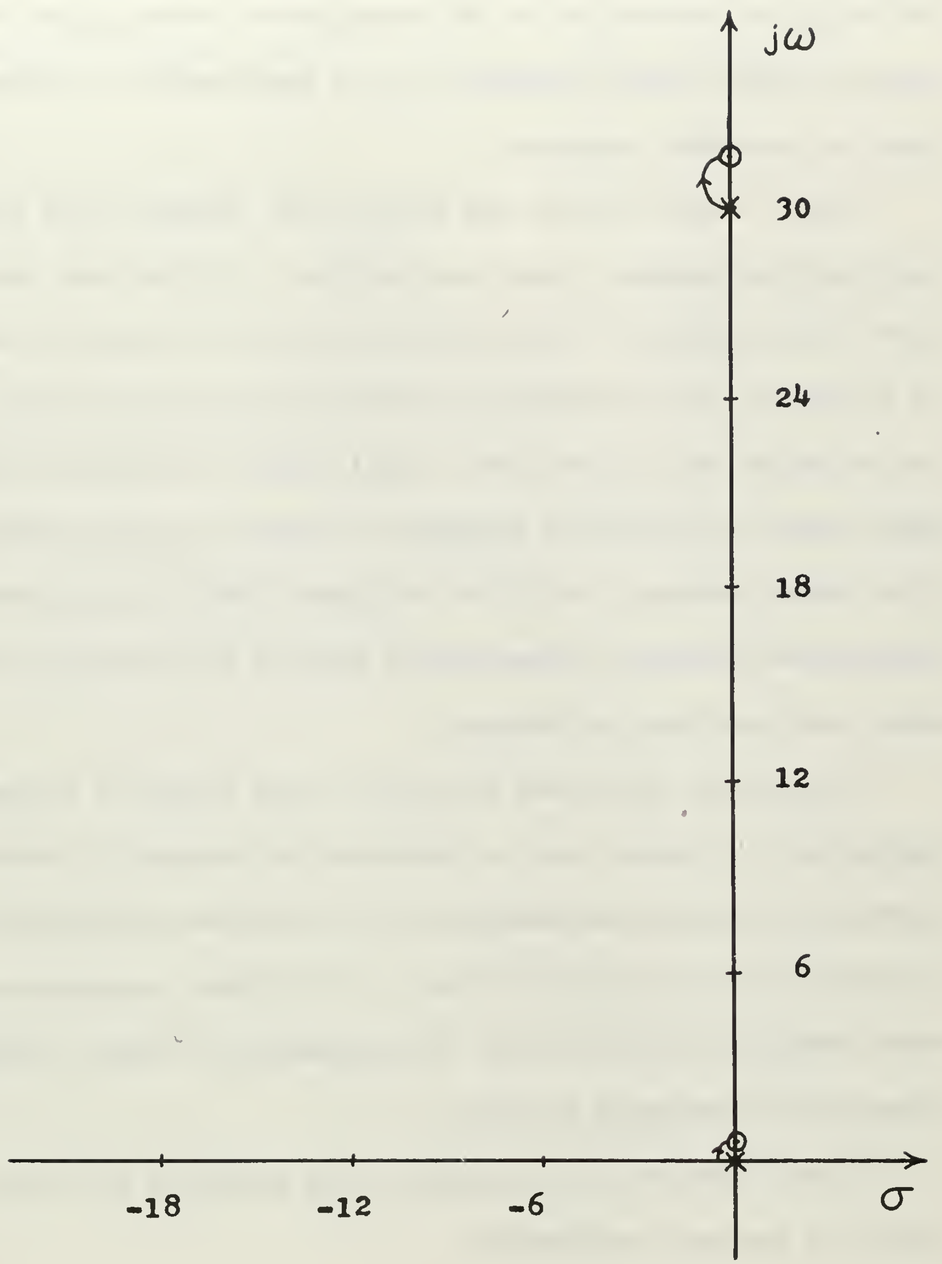


Fig. 49. Root locus of 2^{nd} order system with two sections of cascaded, phase-lag filter.

9. Conclusions.

This paper has investigated the applicability of a root locus stability criterion to the analysis and design of feedback control systems. The stability criterion has the advantage of reducing the order of the equations that must be handled, and is applicable to systems with more than one variable parameter.

Third, fourth, fifth, and sixth order systems, with tachometer and acceleration feedback, have been analyzed. It has been shown that this root locus criterion is an effective tool in the analysis of the effects of tachometer and acceleration feedback on system stability. The stability criterion may be used for a simple check to determine whether tachometer and/or acceleration feedback is capable of stabilizing fifth and sixth order systems. For fifth and sixth order systems, conditions have been derived defining systems which may not be stabilized with tachometer and acceleration feedback.

It has been shown that this root locus stability criterion is a particularly valuable tool in analyzing and designing tachometer and acceleration feedback compensation for third order systems and below. It is useful in analyzing the effects of this type compensation on a fourth order system, and may be used for compensation design, providing that second order dominance is shown.

It was found that, in general, this method is not useful in the design of cascade compensation.

In conclusion, this root locus stability criterion is most useful for the analysis of systems with two variable parameters, in which one parameter occurs only in the even part of the characteristic equation, and the other parameter occurs only in the odd part. Under these conditions the stability criterion will generally be an effective design tool

for fourth order systems and below. On an individual basis, it may be useful in the design of systems with order greater than four.

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13. ABSTRACT

Root locus techniques are used infrequently in the design of tachometer and acceleration feedback compensation for feedback control systems. A root locus stability criterion is discussed, which has the capability of handling more than one variable coefficient in the characteristic equation. This stability criterion is applied to the analysis and design of tachometer and acceleration feedback compensation. The design of cascade compensation is attempted, and the difficulties of designing this type of compensation with the root locus technique are discussed. The root locus stability criterion is found to be a useful tool for designing tachometer and acceleration feedback compensation for third order systems, and for certain fourth order systems.

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